Grade 10 Mathematics Lesson Plan

Probability Using Tree Diagrams

Strand:	**Statistics and Probability**
Sub-Strand:	Probability 1: Probability Using Tree Diagrams
Specific Learning Outcome:	Determine the probability of independent events using tree diagrams
Duration:	40 minutes
Key Inquiry Questions:	How is probability applied in real life situations?
Learning Resources:	CBC Grade 10 textbooks, colored markers, chart paper, coins, marbles or balls

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Drawing Balls Without Replacement

Objective: Students work in pairs to construct a tree diagram step by step, discovering how probabilities change when sampling without replacement.

Work in pairs to complete the following tasks:

Scenario: A bag contains 5 green balls and 3 red balls, making 8 balls in total. You will pick three balls one at a time without putting any back.

Task (a): Draw the First Stage

Draw a starting point and create two branches to show the two possible colours that could be selected first. Label each branch with the probability of selecting that colour based on the number of balls in the bag at the beginning.

Task (b): Draw the Second Stage from Red Branch

From the branch that represents selecting a red ball first, draw two new branches to show the possible outcomes for the second pick. Think carefully about how the total number of balls and the number of red balls have changed after removing one red ball, and label the branches using the new probabilities.

Task (c): Draw the Third Stage

From the branch that represents selecting red on both the first and second picks, draw the final set of branches for the third pick. Determine how many balls remain in total and how many red balls are still available, and use this information to label the branches correctly.

Task (d): Trace the Path

Trace the path that represents picking a red ball in all three draws. Describe what happens to the numerators and denominators of the probabilities as you move along this path and explain why these changes occur.

Task (e): Calculate the Probability

Use the rule for finding the probability of successive events by multiplying the probabilities along a single branch. Write out the multiplication clearly and simplify your result.

Discussion prompts for teachers:

- How did you label the first branches? What probabilities did you use?
- After removing one red ball, how many balls are left? How many are red?
- Why do the denominators decrease as we move along the tree?
- What operation did you use to find the final probability? Why multiply?
- Does the order matter? Would green-red-red give a different probability than red-green-red?

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

1. What is a Probability Tree Diagram?

A Probability Tree Diagram is a visual way to organize and calculate probabilities for multiple events happening one after another.

2. Golden Rules for Using Tree Diagrams

Rule 1: Branches from the same point add to 1

The probabilities of all branches originating from a single node must always add up to exactly 1 (or 100%), because they represent all possible outcomes for that specific event.

Rule 2: Multiply along the branches (The "AND" Rule)

To find the probability of a sequence of events (e.g., Event A and then Event B), you multiply the probabilities along the path.

Example: P(Red AND Red) = P(Red first) × P(Red second)

Rule 3: Add down the columns (The "OR" Rule)

To find the overall probability of different paths that lead to a successful outcome (e.g., Path 1 or Path 2), you add their final probabilities together.

Example: P(Same color) = P(Red AND Red) + P(Blue AND Blue)

3. With Replacement vs Without Replacement

- With replacement: Probabilities stay the same at each stage (independent events)
- Without replacement: Probabilities change at each stage (dependent events)

Phase 3: Practice and Application (15 minutes)

Worked Example 3.2.27 (Coin Tosses - Independent)

Problem: A fair coin is tossed twice. Draw a tree diagram to represent this information and use it to find the probability of getting two Heads.

Solution:

Step 1: Identify event type

The events are independent because the result of the first toss does not affect the second toss.

Step 2: Draw the tree diagram

First toss: Two branches (H or T), each with probability 0.5

Second toss: From each branch, two more branches (H or T), each with probability 0.5

Step 3: Find P(H and H)

Trace the path: H on first toss, then H on second toss

Multiply along the path: P(H and H) = $0.5 \times 0.5 = 0.25$

Answer: The probability of getting two Heads is 0.25 or 25% or 1/4

Worked Example 3.2.28 (Marbles - Without Replacement)

Problem: A bag contains 5 Red marbles and 3 Blue marbles. Two marbles are drawn at random without replacement. Draw a tree diagram and find the probability of drawing two marbles of the same color.

Solution:

Step 1: Identify event type

Without replacement - the probabilities change after the first draw.

Step 2: Draw the tree diagram

First draw: P(Red) = 5/8, P(Blue) = 3/8

Second draw (if Red first): P(Red) = 4/7, P(Blue) = 3/7 (only 7 marbles left, 4 red)

Second draw (if Blue first): P(Red) = 5/7, P(Blue) = 2/7 (only 7 marbles left, 2 blue)

Step 3: Calculate P(Same Color)

P(Red, Red) = 5/8 × 4/7 = 20/56

P(Blue, Blue) = 3/8 × 2/7 = 6/56

P(Same Color) = 20/56 + 6/56 = 26/56 = 13/28

Answer: The probability of drawing two marbles of the same color is 13/28 or approximately 46.4%

Phase 4: Assessment (5 minutes)

Exit Ticket

Using tree diagrams, solve:

1. A spinner has 3 equal sections colored Red, Green, and Yellow. You spin it twice. Draw a tree diagram and find the probability that the spinner lands on Red both times.

2. A jar contains 4 orange sweets and 6 lemon sweets. Sarah takes one sweet at random, eats it, and then takes another one. Calculate the probability that she eats two sweets of different flavors.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn tree diagram templates with empty branches to fill in.
- Use actual coins and colored balls for hands-on probability experiments.
- Start with two-stage tree diagrams before moving to three stages.
- Color-code branches (green for first event, blue for second event).
- Provide step-by-step calculation guides: "Multiply along, add down".
- Work in pairs with peer support.
- Use fraction calculators to check multiplication.

For Advanced Students:

- Explore three or more stages in tree diagrams.
- Calculate probabilities involving "at least one" outcomes using complements.
- Investigate conditional probability using tree diagrams.
- Compare theoretical probabilities from tree diagrams with experimental results.
- Create their own word problems involving tree diagrams.

- Explore real-world applications: genetics (Punnett squares), quality control, medical testing.

Extension Activity: Real-World Tree Diagrams

Scenario: Use tree diagrams to solve a real-world conditional probability problem.

Problem: A farmer in Eldoret is planting maize. The probability that there will be adequate rain this season is 0.7. If it rains, the probability of a good harvest is 0.8. If it does not rain, the probability of a good harvest drops to 0.2. Calculate the probability that the farmer will have a good harvest.

Tasks:

1. Draw a tree diagram with two stages: Rain (Yes/No) and Harvest (Good/Poor).
2. Label the first stage branches: P(Rain) = 0.7, P(No Rain) = 0.3.
3. Label the second stage branches from Rain: P(Good|Rain) = 0.8, P(Poor|Rain) = 0.2.
4. Label the second stage branches from No Rain: P(Good|No Rain) = 0.2, P(Poor|No Rain) = 0.8.
5. Identify all paths that lead to a Good Harvest.
6. Calculate: P(Rain AND Good) = 0.7 × 0.8 = 0.56.
7. Calculate: P(No Rain AND Good) = 0.3 × 0.2 = 0.06.
8. Add the probabilities: P(Good Harvest) = 0.56 + 0.06 = 0.62 or 62%.
9. Discussion: How does rain affect the harvest probability? What if rain probability was only 0.4?