

# Step by step guide: Quadratic Identities

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## Grade 10 Mathematics | 40-Minute Lesson

### Before Class Begins

#### Preparation Checklist:

- Write the three identities on the board (covered until Phase 2)
- Prepare group discussion prompts
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have worked examples ready

### PHASE 1: Problem-Solving and Discovery (15 Minutes)

#### Opening (2 minutes)

##### [SAY]:

*"Good morning/afternoon, class! Today we're going to discover some powerful shortcuts in algebra called QUADRATIC IDENTITIES. These are special formulas that make expanding and factoring much faster!"*

##### [SAY]:

*"Here's our key question: How do we apply the concept of quadratic equations? Let's explore together."*

#### Anchor Activity Introduction (3 minutes)

##### [SAY]:

*"Form groups of at least 4 people. Your task has two parts:*

*Part 1: Define and discuss these terms:*

- 1. Quadratic identities*
- 2. Difference of squares*
- 3. Perfect squares*
- 4. Factorization of quadratic expressions*

Part 2: Copy and observe these three identities:

(i)  $(a + b)^2 = a^2 + 2ab + b^2$

(ii)  $(a - b)^2 = a^2 - 2ab + b^2$

(iii)  $(a - b)(a + b) = a^2 - b^2$

### Group Work (7 minutes)

**[SAY]:**

*"Discuss in your groups:*

- *Can you verify these identities by expanding?*
- *How do the identities help us solve expressions faster?*
- *What real-world applications might use these identities?*

*You have 6 minutes. Begin!"*

**[DO]:** Walk around the room, observing group discussions.

**[ASK probing questions as you circulate]:**

- "Can you expand  $(a + b)^2$  to verify the identity?"
- "What do you notice about the middle term?"
- "Why is  $(a - b)(a + b)$  called the difference of squares?"
- "What happens to the middle terms when you expand  $(a - b)(a + b)$ ?"
- "How is  $(a + b)^2$  different from  $(a - b)^2$ ?"

**[TIME CHECK]:** At 5 minutes, announce: "One more minute!"

### Class Discussion (3 minutes)

**[SAY]:**

*"Let's share what you discovered. What is an identity?"*

**[Expected answer]:** "An equation that is true for all values of the variable."

**[ASK]:**

*"When you expanded  $(a - b)(a + b)$ , what happened to the middle terms?"*

**[Expected answer]:** "They cancelled out! We got  $-ab + ab = 0$ "

**[TRANSITION]:**

*"Excellent! Let me formalize these three powerful identities."*

## PHASE 2: Structured Instruction (10 Minutes)

### The Three Quadratic Identities (6 minutes)

[REVEAL identities on board]:

[SAY]:

*"Quadratic identities are special formulas that simplify our work with quadratic expressions. There are THREE essential identities:"*

[WRITE Identity 1]:

*"Identity 1: Perfect Square (Sum)*

$$(a + b)^2 = a^2 + 2ab + b^2$$

*The middle term is POSITIVE and equals TWICE the product of a and b."*

[WRITE Identity 2]:

*"Identity 2: Perfect Square (Difference)*

$$(a - b)^2 = a^2 - 2ab + b^2$$

*The middle term is NEGATIVE."*

[WRITE Identity 3]:

*"Identity 3: Difference of Squares*

$$(a + b)(a - b) = a^2 - b^2$$

*There is NO middle term! The middle terms cancel out."*

### Factoring with Difference of Squares (2 minutes)

[SAY]:

*"The difference of squares works BOTH ways:*

*Expanding:  $(a + b)(a - b) = a^2 - b^2$*

*Factoring:  $a^2 - b^2 = (a + b)(a - b)$*

*When you see  $a^2 - b^2$ , you can immediately factor it!"*

### Addressing Misconceptions (2 minutes)

[SAY - IMPORTANT]:

"COMMON MISTAKE:  $(a + b)^2$  is NOT equal to  $a^2 + b^2$ !

You *MUST* include the middle term  $2ab$ !

Let me prove it:  $(3 + 2)^2 = 5^2 = 25$

But  $3^2 + 2^2 = 9 + 4 = 13 \neq 25$

The correct answer:  $3^2 + 2(3)(2) + 2^2 = 9 + 12 + 4 = 25 \checkmark$  "

**[TRANSITION]:**

"Now let's practice using these identities!"

### PHASE 3: Practice and Application (15 Minutes)

#### Worked Example (4 minutes)

**[SAY]:**

"Let's factor  $x^2 - 16$  using the difference of squares."

**[WRITE step by step]:**

"Step 1: Recognize this is a difference of squares

$x^2 - 16$  is in the form  $a^2 - b^2$

Step 2: Identify  $a$  and  $b$

$a^2 = x^2$ , so  $a = x$

$b^2 = 16$ , so  $b = 4$

Step 3: Apply the identity

$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$ "

#### Guided Practice (5 minutes)

**[SAY]:**

"Try these with your partner:

a) Factor:  $9y^2 - 25$

b) Expand:  $(2x + 3)^2$ "

**[GIVE 4 minutes, then review]:**

$$\begin{aligned} & \text{"a) } 9y^2 - 25 \\ & = (3y)^2 - 5^2 \\ & = (3y - 5)(3y + 5) \end{aligned}$$

$$\begin{aligned} & \text{b) } (2x + 3)^2 \\ & = (2x)^2 + 2(2x)(3) + 3^2 \\ & = 4x^2 + 12x + 9 \end{aligned}$$

### Independent Practice (6 minutes)

**[SAY]:**

*"Now try these on your own:*

a) Factor:  $x^2 - 49$

b) Factor:  $x^2 + 6x + 9$

c) Expand:  $(3y - 2)^2$ "

**[GIVE 5 minutes, then quickly check]:**

a)  $(x - 7)(x + 7)$

b)  $(x + 3)^2$  [This is a perfect square trinomial!]

c)  $9y^2 - 12y + 4$ "

**[TRANSITION]:**

*"Now I want to see what each of you has learned."*

### PHASE 4: Assessment / Checkpoint (8 Minutes)

#### Independent Work (5 minutes)

**[DISPLAY questions]:**

1. Factor:  $4a^2 - 9$

2. Expand:  $(x + 7)^2$

3. A square garden has sides of length  $(x + 5)$  meters. Write an expression for the area."

**[SAY]:**

*"You have 5 minutes. Begin."*

## Collection and Closure (2 minutes)

[SAY]:

*"Time's up. Please pass your exit tickets forward."*

[COLLECT all tickets]

[SAY]:

*"Today you learned the THREE quadratic identities:*

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$

*Remember: Always include the middle term in perfect squares!"*

[SAY]:

*"Great work today! For homework, practice more factoring problems."*

## Differentiation Notes

**For Struggling Learners:**

- Provide identity reference cards
- Use color coding to highlight a, b, and the middle term
- Start with numerical examples (e.g.,  $5^2 - 3^2 = (5-3)(5+3)$ )
- Allow peer support during practice

**For Advanced Learners:**

[GIVE these extensions]:

- Factor  $x^4 - 16$  completely
- Prove that  $(a+b)^2 - (a-b)^2 = 4ab$
- Use difference of squares to calculate  $51 \times 49$  mentally

## Answer Key

**Exit Ticket Answers:**

1.  $4a^2 - 9: (2a - 3)(2a + 3)$

2.  $(x + 7)^2: x^2 + 14x + 49$

3. **Area of garden:**  $(x + 5)^2 = x^2 + 10x + 25$  square meters

### Extension Answers:

1.  $x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$

2.  $(a+b)^2 - (a-b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$

3.  $51 \times 49 = (50+1)(50-1) = 50^2 - 1 = 2500 - 1 = 2499$

### Post-Lesson Reflection Prompts

1. **What went well?** Did students recognize the patterns in the identities?
2. **What would I change?** Was the misconception about  $(a+b)^2$  addressed effectively?
3. **Student Understanding:** Could students distinguish between the three identities?
4. **Next Steps:** Which students need more practice with factoring?