Step by step guide: Zero and Negative Indices

Grade 10 Mathematics | 40-Minute Lesson

Before Class Begins

Preparation Checklist:

- Write the pattern table on the board ($2^4 = 16$, $2^3 = 8$, etc.)
- Ensure each student has pen, paper, and calculator
- Prepare exit tickets for distribution
- Set timer for phase transitions
- Have the two rules ready to reveal (covered until Phase 2)

PHASE 1: Problem-Solving and Discovery (15 Minutes)

Opening (2 minutes)

[SAY]:

"Good morning/afternoon, class! We've been learning about indices and their laws. Today, we're going to explore what happens when the exponent is ZERO or NEGATIVE. This might seem strange at first, but you'll discover it makes perfect sense!"

[SAY]:

"Here's our key question: How do we use real numbers in day-to-day activities? Let's discover the answer through a pattern."

Anchor Activity Introduction (2 minutes)

[POINT to the board]:

[SAY]:

"Look at this pattern on the board:
$2^4 = 16$
$2^3 = 8$

$2^2 = 4$
$2^1 = 2$
$2^0 = ?$"

[ASK]:

"What pattern do you notice as the exponent decreases by 1?"

[WAIT for responses]: Expected: "The value is divided by 2 each time."

Individual/Pair Exploration (8 minutes)

[SAY]:

"Now I want you to:
1. Continue the pattern to find 2^0
2. Predict what 2^{-1} and 2^{-2} would be
3. Try the same pattern with 3^n
4. Discuss with your partner what you discover

You have 6 minutes. Begin!"

[DO]: Walk around the room, observing student work.

[ASK probing questions as you circulate]:

- "What operation connects 16 to 8? And 8 to 4?"
- "If we keep dividing by 2, what comes after 2?"
- "What is 2 divided by 2?"
- "If $2^0 = 1$, what would 2^{-1} be?"
- "Does your pattern work for base 3 as well?"

[TIME CHECK]: At 6 minutes, announce: "Two more minutes!"

Class Discussion (3 minutes)

[SAY]:

"Let's share what you discovered. What is 2^0?"

[Expected answer]: "1"

[ASK]:

"Why does that make sense?"

[Expected answer]: "Because 2 ÷ 2 = 1"

[ASK]:

"What about 2^{-1}?"

[Expected answer]: "1/2 or 0.5"

[TRANSITION]:

"Excellent! You've discovered two important rules. Let me formalize them."

PHASE 2: Structured Instruction (10 Minutes)

Rule 1: Zero Index (4 minutes)

[WRITE on board]:

RULE 1: ZERO INDEX

[SAY]:

"Any non-zero number raised to the power zero equals 1.

$a^0 = 1$ (where $a \neq 0$)

Examples:
- *$5^0 = 1$*
- *$100^0 = 1$*
- *$(-3)^0 = 1$"*

[EXPLAIN why]:

"This makes sense because of the quotient law:
$a^n \div a^n = a^{n-n} = a^0$
But $a^n \div a^n = 1$
Therefore, $a^0 = 1$"

Rule 2: Negative Index (4 minutes)

[WRITE on board]:

RULE 2: NEGATIVE INDEX

[SAY]:

"A negative exponent means taking the reciprocal of the base.

$a^{-n} = 1/a^n$

Examples:
- *$2^{-1} = 1/2^1 = 1/2$*
- *$3^{-2} = 1/3^2 = 1/9$*
- *$5^{-3} = 1/5^3 = 1/125$"*

Addressing Misconceptions (2 minutes)

[SAY - IMPORTANT]:

"Two common mistakes to avoid:

1. 0^0 is UNDEFINED—the zero index rule only works when the base is NOT zero.

2. A negative index does NOT make the answer negative!
$2^{-3} = 1/8$ (positive!), NOT −8"

[TRANSITION]:

"Now let's apply these rules to real-world problems!"

PHASE 3: Practice and Application (15 Minutes)

Guided Practice (5 minutes)

[SAY]:

"Let's work through some problems together."

[PROBLEM 1 - Carpenter's Plank]:

"A carpenter cuts wooden planks. Each cut halves the length.
- *Original: $2^0 = 1$ metre*
- *After 1 cut: $2^{-1} = 1/2$ metre*
- *After 2 cuts: $2^{-2} = 1/4$ metre*
- *After 3 cuts: $2^{-3} = 1/8$ metre*
- *After 4 cuts: $2^{-4} = 1/16$ metre"*

[ASK]:

"What formula represents the length after n cuts?"

[Expected answer]: "$L = 2^{-n}$"

Quick Practice (5 minutes)

[PROBLEM 2]:

"Simplify: $8b^0$"

[WAIT, then solve]:

"$8b^0 = 8 \times 1 = 8$"

[PROBLEM 3]:

"Simplify without negative indices: $6b^{-1}$"

[WAIT, then solve]:

"$6b^{-1} = 6 \times (1/b) = 6/b$"

Partner Practice (5 minutes)

[SAY]:

"Work with your partner:
a) Simplify: $5x^0 + 3$
b) Write without negative indices: $4a^{-2}$
c) If 64 books are halved each day, how many after 3 days?"

[GIVE 3 minutes, then review]:

"a) $5x^0 + 3 = 5(1) + 3 = 8$
b) $4a^{-2} = 4/a^2$
c) $64 \times 2^{-3} = 64 \times (1/8) = 8$ books"

[TRANSITION]:

"Now I want to see what each of you has learned."

PHASE 4: Assessment / Checkpoint (8 Minutes)

Independent Work (5 minutes)

[DISPLAY questions]:

"1. A library has 64 books. Half are removed each day. How many remain after 5 days? Use indices.

2. A patient takes 400 mg of medicine. Every 4 hours, it reduces to half. How much remains after 12 hours?"

[SAY]:

"You have 5 minutes. Begin."

Collection and Closure (2 minutes)

[SAY]:

"Time's up. Please pass your exit tickets forward."

[COLLECT all tickets]

[SAY]:

"Today you learned two important rules:
• $a^0 = 1$ (any non-zero number to the power zero equals 1)
• $a^{-n} = 1/a^n$ (negative index means reciprocal)

These rules help us model real-world situations like depreciation, decay, and halving!"

[SAY]:

"Great work today! For homework, complete the remaining problems from the exit ticket."

Differentiation Notes

For Struggling Learners:

- Provide the pattern table pre-filled up to 2^1
- Use visual aids showing halving (folding paper)
- Start with base 2 before other bases
- Allow calculator use for verification

For Advanced Learners:

[GIVE this extension]:

"A school installs 100 energy-saving bulbs. Every year, a quarter stop working.
a) Write an index expression for working bulbs after 4 years.
b) How many bulbs are functional after 6 years?"

Answer Key

Exit Ticket Answers:

1. Library books after 5 days: $64 \times 2^{-5} = 64 \times (1/32) = 2$ books

2. Medicine after 12 hours (3 intervals): $400 \times 2^{-3} = 400 \times (1/8) = 50$ mg

Additional Assessment Answers:

Van value after 9 years: $1{,}000{,}000 \times 2^{-3}$ = Ksh 125,000

Van value after 15 years: $1{,}000{,}000 \times 2^{-5}$ = Ksh 31,250

Medicine after 20 hours: $400 \times 2^{-5} = 12.5$ mg

Energy bulbs after 4 years: $100 \times (3/4)^4 \approx 32$ bulbs

Energy bulbs after 6 years: $100 \times (3/4)^6 \approx 18$ bulbs

Post-Lesson Reflection Prompts

1. What went well? Did the pattern discovery help students understand the rules?

2. What would I change? Was enough time given for exploration?

3. Student Understanding: Did students confuse negative index with negative answer?

4. Next Steps: Which students need more practice with real-world applications?