CBC Grade 10 Mathematics Lesson Plan

Logarithms of Powers and Roots

Strand	Numbers and Algebra
Sub-Strand	Indices and Logarithms
Specific Learning Outcome	Apply common logarithms in multiplication, division, powers and roots of numbers
Key Inquiry Questions	How do we use real numbers in day-to-day activities?
Learning Resources	CBC Grade 10 textbooks, scientific calculators, logarithm tables
Lesson Duration	40 minutes

Lesson Structure Overview

Phase	Activity	Duration
Phase 1	Problem-Solving and Discovery (Anchor Activity)	15 minutes
Phase 2	Structured Instruction (Key Takeaways)	10 minutes
Phase 3	Practice and Application (Worked Examples)	15 minutes
Phase 4	Assessment (Exit Ticket)	5 minutes

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Exploring Logarithms of Powers and Roots

Material needed:

- • Pen and paper
- • Scientific calculator

Instructions:

Use your calculator and math reasoning to solve the following expressions involving logarithms:

1. 1. Calculate log(10000)

Now rewrite 10000 as a power of 10: $\log(10000) = \log(10^4)$

Apply the logarithmic law: $\log(a^b) = b \times \log(a)$

$\log(10^4) = 4 \times \log(10) = 4 \times 1 = 4$

2. 2. Try the following:

- • $\log(100)$
- • $\log(1000000)$
- • $\log(\sqrt{100})$
- • $\log(\sqrt[3]{1000})$

Discussion Questions:

- • What do you observe when applying logarithms to square and cube roots?
- • Why does $\log(\sqrt{100})$ give half of $\log(100)$?
- • What general rule can you form for logarithms and powers?

Teacher Guidance for Anchor Activity

This anchor activity helps students discover the power and root laws of logarithms through exploration with calculators. Students will observe patterns and formulate rules before formal instruction.

Facilitation Strategy:

- • Allow students to use calculators freely to explore
- • Encourage students to express numbers as powers of 10
- • Ask probing questions: "What pattern do you notice?" "How does the exponent relate to the logarithm?"
- • Guide students to see that $\log(a^b) = b \times \log(a)$
- • Help students recognize that roots are fractional powers ($\sqrt{a} = a^{(1/2)}$)
- • Use student discoveries as a bridge to formal logarithm laws

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

After students have explored through the anchor activity, formalize their discoveries with these key concepts:

1. Logarithms Simplify Powers and Roots

Logarithms help simplify calculations involving powers and roots by converting multiplication and division into addition and subtraction, and powers and roots into multiplication and division.

2. Logarithm of a Power

When you take the logarithm of a number raised to a power, you can bring the exponent down as a multiplier:

$\log(a^b) = b \times \log(a)$

Examples:

- • $\log(10^4) = 4 \times \log(10) = 4 \times 1 = 4$
- • $\log(2^3) = 3 \times \log(2) = 3 \times 0.3010 = 0.9030$
- • $\log(5^6) = 6 \times \log(5) = 6 \times 0.6990 = 4.194$

3. Logarithm of a Root

Roots can be expressed as fractional powers. For example, $\sqrt{a} = a^{(1/2)}$ and $\sqrt[3]{a} = a^{(1/3)}$.

Therefore:

$\log(\sqrt{a}) = \log(a^{(1/2)}) = (1/2) \times \log(a)$

$\log(\sqrt[3]{a}) = \log(a^{(1/3)}) = (1/3) \times \log(a)$

Examples:

- • $\log(\sqrt{100}) = (1/2) \times \log(100) = (1/2) \times 2 = 1$
- • $\log(\sqrt[3]{1000}) = (1/3) \times \log(1000) = (1/3) \times 3 = 1$

4. Why This Works

The power law works because logarithms are exponents. When we raise a number to a power and then take the logarithm, we are essentially counting how many times we multiply the base. The exponent tells us how many times to repeat this process.

5. Practical Applications

These laws allow us to:

- • Calculate large powers without direct multiplication
- • Find roots of numbers using logarithm tables
- • Solve exponential equations
- • Simplify complex calculations in science and engineering

Scaffolding Strategies

Address common misconceptions revealed during the anchor activity:

- • Clarify that the exponent "comes down" as a multiplier
- • Emphasize that roots are fractional powers
- • Show the connection: $\sqrt{a} = a^{(1/2)}$, $\sqrt[3]{a} = a^{(1/3)}$
- • Use visual representations to show repeated multiplication
- • Connect to prior knowledge of indices

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Evaluating a Power Using Logarithms

Evaluate $(23.5)^4$ using logarithm tables.

Solution:

Step 1: Take the logarithm of both sides

Let $x = (23.5)^4$

$\log(x) = \log((23.5)^4)$

Step 2: Apply the power law

$\log(x) = 4 \times \log(23.5)$

Step 3: Find log(23.5) from the logarithm table

$\log(23.5) = 1.3711$

Step 4: Multiply by the exponent

$\log(x) = 4 \times 1.3711 = 5.4844$

Step 5: Find the antilogarithm

$x = \text{antilog}(5.4844) = 304{,}000$

Answer: $(23.5)^4 = 304{,}000$

Example 2: Finding a Cube Root Using Logarithms

Evaluate $\sqrt[3]{524.8}$ using logarithm tables.

Solution:

Step 1: Express as a fractional power

$\sqrt[3]{524.8} = (524.8)^{(1/3)}$

Step 2: Take the logarithm

$\log(\sqrt[3]{524.8}) = \log((524.8)^{(1/3)})$

Step 3: Apply the power law

$\log(\sqrt[3]{524.8}) = (1/3) \times \log(524.8)$

Step 4: Find log(524.8) from the table

$\log(524.8) = 2.7200$

Step 5: Divide by 3

$\log(\sqrt[3]{524.8}) = 2.7200 \div 3 = 0.9067$

Step 6: Find the antilogarithm

$\sqrt[3]{524.8} = \text{antilog}(0.9067) = 8.065$

Answer: $\sqrt[3]{524.8} \approx 8.065$

Example 3: Computing $(78.5)^3$ Using Logarithms

Compute $(78.5)^3$ using logarithms.

Solution:

Step 1: Let $x = (78.5)^3$, then $\log(x) = \log((78.5)^3)$

Step 2: Apply power law: $\log(x) = 3 \times \log(78.5)$

Step 3: From tables: $\log(78.5) = 1.8949$

Step 4: Multiply: $\log(x) = 3 \times 1.8949 = 5.6847$

Step 5: Find antilog: $x = \text{antilog}(5.6847) = 484{,}000$

Answer: $(78.5)^3 \approx 484{,}000$

Individual Practice (Students work independently)

Provide students with similar problems to solve:

3. 1. Evaluate $(12.4)^5$ using logarithms
4. 2. Find $\sqrt{2500}$ using logarithms
5. 3. Calculate $(254.6)^4$ using logarithms
6. 4. Determine $\sqrt[3]{8000}$ using logarithms

Phase 4: Assessment - Exit Ticket (5 minutes)

Students complete individually to demonstrate understanding:

Question 1: The volume of a cube is 79,507 cubic centimeters. Use logarithms to find the length of one side.

(Hint: Volume = side^3, so side = $\sqrt[3]{\text{Volume}}$)

Question 2: Compute $(78.5)^3$ using logarithms.

Question 3: Compute $(254.6)^4$ using logarithms.

Exit Ticket Answer Key

Question 1:

Let side = s, then $s^3 = 79{,}507$

$s = \sqrt[3]{79{,}507}$

$\log(s) = (1/3) \times \log(79{,}507)$

$\log(79{,}507) = 4.9004$

$\log(s) = 4.9004 \div 3 = 1.6335$

$s = \text{antilog}(1.6335) = 43.0$ cm

Question 2:

$\log((78.5)^3) = 3 \times \log(78.5) = 3 \times 1.8949 = 5.6847$

$(78.5)^3 = \text{antilog}(5.6847) \approx 484{,}000$

Question 3:

$\log((254.6)^4) = 4 \times \log(254.6) = 4 \times 2.4059 = 9.6236$

$(254.6)^4 = \text{antilog}(9.6236) \approx 4{,}200{,}000{,}000$

Differentiation Strategies

For Struggling Learners:

- • Provide step-by-step formula cards
- • Start with simple powers of 10
- • Use visual aids showing the power law
- • Allow extra time with calculators
- • Practice converting roots to fractional powers
- • Provide worked examples as reference

For Advanced Learners:

- • Introduce higher order roots (4th, 5th roots)
- • Combine power law with product and quotient laws
- • Solve exponential equations using logarithms
- • Explore applications in compound interest
- • Investigate logarithmic scales (Richter, pH, decibels)
- • Challenge with problems involving multiple operations

Extension Activity

Real-World Applications Investigation

Objective: Apply logarithms of powers and roots to real-world scenarios.

Activity Description:

7. 1. Research one real-world application (choose one):

- • Compound interest calculations (money growing exponentially)
- • Population growth models
- • Radioactive decay in nuclear physics
- • Sound intensity (decibel scale)
- • Earthquake magnitude (Richter scale)

8. 2. Identify how powers and roots are involved.
9. 3. Use logarithms to solve a related problem.
10. 4. Present findings with calculations to the class.

Calculator vs. Tables Challenge

Students compare results:

- • Solve the same power/root problem using both calculator and logarithm tables
- • Compare accuracy and efficiency
- • Discuss when each method is more appropriate
- • Reflect on the historical importance of logarithm tables

Post-Lesson Reflection for Teachers

- • Did students successfully apply the power law: $\log(a^b) = b \times \log(a)$?
- • Were students able to convert roots to fractional powers?
- • What misconceptions emerged during the lesson?
- • How comfortable were students using logarithm tables?
- • Did the anchor activity effectively introduce the concept?
- • What adjustments are needed for future lessons on this topic?