

Grade 10 Mathematics Lesson Plan

Volume of Spheres

Strand:	Measurement and Geometry
Sub-Strand:	Volume: Volume of a Sphere
Specific Learning Outcome:	Calculate the volume of prisms, pyramids, cones, frustums and spheres. Explore the use of the surface area and volume of solids in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, oranges (or round fruits), knife, flat surface, calculators, worksheets

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Orange Peeling Experiment
Structured Instruction	10 minutes	Formalizing the sphere volume formula
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Orange Peeling Experiment

Objective: Students will discover the formula for finding the volume of a sphere by peeling oranges, flattening the peels, and exploring how the radius affects volume.

Materials Needed:

- An orange (or any round fruit) - one per group
- A knife
- A flat surface

- Calculators
- Worksheets for recording observations and calculations

Activity Steps:

1. Step 1: Cut the orange in half and carefully peel the skin off in small sections.
2. Step 2: Try flattening the peels and arrange them to see how they approximate a circle area.
3. Step 3: Observe: When you peel an orange and flatten the pieces, you can see that the peels cover a large area. This helps visualize why increasing the radius increases the overall amount of space the fruit takes up (its volume).
4. Step 4: Cutting the Orange into Sections. If you cut an orange in half, you can see its cross-section. If you keep slicing it into smaller spheres, their individual radii determine their volumes.
5. Step 5: Pyramidal Stack of Oranges. Why do you think oranges or tomatoes or apples are stacking in pyramidal stacks in the market? Oranges in a fruit market are often packed in pyramidal stacks because spheres fit together efficiently. The larger the radius, the more space each orange occupies, which directly affects storage and packaging.

Discussion Questions:

6. What shape is a sphere?
7. What happens to the volume when the radius doubles?
8. Why are oranges stacked in pyramidal stacks in the market?
9. How does the radius affect the volume of a sphere?
10. Can you think of real-world examples of spheres?

Teacher Role During Discovery:

- Circulate among groups, ensuring students peel oranges safely.
- Ask probing questions: What happens when you flatten the peels? How much area do they cover?
- For struggling groups: Let us peel together. What do you notice about the area?
- For early finishers: If the radius doubles, how does the volume change?
- Guide students to articulate: The radius is the most important factor in determining the volume of a sphere. If the radius doubles, the volume increases by 2 cubed equals 8 times!
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Sphere Volume Formula

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the volume of a sphere.

Key Takeaway: What is a Sphere?

A sphere is a perfectly round object. Every point on the surface of a sphere is the same distance from the center. Examples include balls, oranges, planets, and bubbles.

Formula:

Volume of a Sphere = $(4 / 3)$ times π times r cubed

Where:

- r is the radius of the sphere
- π approximately equals 3.142 or $22 / 7$

Key Insight:

The radius is the most important factor in determining the volume of a sphere. If the radius doubles, the volume increases by 2 cubed equals 8 times! This explains why a slightly bigger orange holds significantly more juice compared to a smaller one.

Scaffolding Strategies to Address Misconceptions:

- Misconception: A sphere is the same as a circle. Clarification: No, a circle is two-dimensional (flat), while a sphere is three-dimensional (round).
- Misconception: The volume formula is πr cubed. Clarification: No, you must multiply by $(4 / 3)$. The formula is $(4 / 3) \pi r$ cubed.
- Misconception: If the radius doubles, the volume doubles. Clarification: No, if the radius doubles, the volume increases by 8 times (2 cubed equals 8).
- Misconception: I can use the diameter instead of the radius. Clarification: No, the formula uses the radius. If you have the diameter, divide by 2 to get the radius.

Phase 3: Practice and Application (10 minutes)

Worked Examples:

Example 1: A sphere has a radius of 6 cm. Find its volume.

Solution:

$$\begin{aligned}V &= \left(\frac{4}{3}\right) \text{ times } \pi \text{ times } r \text{ cubed} \\&= 3.142 \text{ times } \left(\frac{4}{3}\right) \text{ times } (6) \text{ cubed} \\&= \left(\frac{864}{3}\right) \text{ times } 3.142 \\&= 904.9 \text{ cm cubed}\end{aligned}$$

Example 2: A football has a radius of 9 cm. What is the volume of the ball?

Solution:

$$\begin{aligned}V &= \left(\frac{4}{3}\right) \text{ times } \pi \text{ times } r \text{ cubed} \\&= 3.142 \text{ times } \left(\frac{4}{3}\right) \text{ times } (9 \text{ cm}) \text{ cubed} \\&= 3054.02 \text{ cm cubed}\end{aligned}$$

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A solid sphere has a radius of 7 cm. Find its volume.
2. A bowl is in the shape of a hemisphere with a diameter of 12 cm. Find the volume of the bowl.
3. A sphere has a volume of 500 cm cubed. If the radius is doubled, what will be the new volume?

Answer Key:

1. $V = (4 / 3) \text{ times } \pi \text{ times } 7 \text{ cubed} = 1436.76 \text{ cm cubed.}$
2. Hemisphere volume = $(2 / 3) \text{ times } \pi \text{ times } r \text{ cubed. Radius} = 6 \text{ cm. } V = (2 / 3) \text{ times } 3.142 \text{ times } 6 \text{ cubed} = 452.39 \text{ cm cubed.}$
3. If radius doubles, volume increases by 8 times. New volume = 500 times 8 = 4000 cm cubed.

Differentiation Strategies

For Struggling Learners:

- Provide pre-cut oranges with labeled radii.
- Use spheres with simple dimensions for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the formula into steps: Calculate r cubed, multiply by π , multiply by 4, divide by 3.

For On-Level Learners:

- Encourage students to verify their formula with orange peeling.
- Ask students to explain the relationship between radius and volume.
- Provide mixed practice with different dimensions.
- Challenge students to find the volume when only the diameter is given.

For Advanced Learners:

- Challenge students to derive the formula using integration.
- Explore real-world applications: sports balls, planets, bubbles, water tanks.
- Investigate the relationship between radius and volume (cubic relationship).
- Apply the concept to hemispheres (half-spheres).
- Solve optimization problems: Given a fixed volume, what radius minimizes the surface area?

Extension Activity

Real-World Application: Raindrop Volume Investigation

Work in groups

Situation: A raindrop is modeled as a sphere with a radius of 0.2 cm. If a storm produces 1,000,000 raindrops, what is the total volume of water in liters?

Tasks:

11. Calculate the volume of one raindrop using the sphere formula.
12. Multiply by 1,000,000 to find the total volume in cubic centimeters.
13. Convert the total volume to liters (1 liter equals 1000 cubic cm).
14. Compare different raindrop sizes: What if the radius was 0.3 cm? 0.4 cm?
15. Investigate: How does the radius affect the total volume of water?
16. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the volume of spheres is used in real-world contexts such as sports (balls), astronomy (planets), meteorology (raindrops), engineering (water tanks), and food (oranges, fruits).

Teacher Reflection Prompts

- Did students successfully peel oranges and observe the area?
- Were students able to discover that doubling the radius increases volume by 8 times?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the cubic relationship between radius and volume?
- What adjustments would improve this lesson for future classes?