Grade 10 Mathematics Lesson Plan

Volume of Frustums

Strand:	Measurement and Geometry
Sub-Strand:	Volume: Volume of a Frustum
Specific Learning Outcome:	Calculate the volume of prisms, pyramids, cones, frustums and spheres. Explore the use of the surface area and volume of solids in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we determine the surface area and volume of solids? Why do we determine the surface area and volume of solids?
Learning Resources:	CBC Grade 10 textbooks, real plastic buckets or flowerpots (frustum-shaped), rulers or measuring tapes, water, measuring jug, calculators, worksheets

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Frustum Volume Lab with bucket measurements
Structured Instruction	10 minutes	Formalizing the frustum volume formula
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Frustum Volume Lab

Objective: Students will discover the formula for finding the volume of a frustum by measuring bucket-shaped containers and calculating their capacity using water measurements.

Materials Needed:

- Calculator
- Rulers or measuring tapes
- Water and a measuring jug
- Real plastic buckets, measuring cups, or flowerpots (frustum-shaped)
- Worksheets for dimensions and calculations

Activity Steps:

1. Step 1: Review the volume formula of a cone and note that the frustum is a cone with the top sliced off.
2. Step 2: Observe real-life frustums: buckets, lampshades, party hats cut short, flower pots, juice glasses, etc.
3. Step 3: Students work in pairs or small groups. They measure: Diameter (then radius) of top opening R, Diameter (then radius) of bottom r, Height of the container h.
4. Step 4: Measure and record the dimensions. Label the top radius R and bottom radius r and the height h in your worksheet. Record all measurements in cm.
5. Step 5: Fill the bucket with water and pour it into a measuring jug to find its actual volume in liters.
6. Step 6: Calculate using the formula and find the volume capacity in liters.
7. Step 7: Compare your measured volume with the calculated volume.

Discussion Questions:

8. What shape is a frustum?
9. How is a frustum different from a cone?
10. What real-world objects have frustum shapes?
11. How can we find the volume of a frustum?
12. How does the measured volume compare with the calculated volume?

Teacher Role During Discovery:

- Circulate among groups, ensuring students measure accurately.
- Ask probing questions: What shape is this? How is it different from a cone?
- For struggling groups: Let us measure together. What is the top radius? What is the bottom radius?
- For early finishers: How would the volume change if the top radius doubled?
- Guide students to articulate: A frustum is a cone with the top sliced off.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Frustum Volume Formula

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the volume of a frustum.

Key Takeaway: What is a Frustum?

A frustum is a cone or pyramid that is cut parallel to its base, removing the top portion. This results in a truncated shape with two parallel bases, one smaller than the other. Examples include lampshades, buckets, flower pots, and juice glasses.

Types of Frustums:

- Frustum of a cone: A cone with the top sliced off parallel to the base.
- Frustum of a pyramid: A pyramid with the top sliced off parallel to the base.

Formula:

Volume of a Frustum = (1 / 3) times π times h times (r squared + R squared + R r)

Where:

- r is the radius of the smaller (top) base
- R is the radius of the larger (bottom) base
- h is the height of the frustum
- π approximately equals 3.14 or 22 / 7

Scaffolding Strategies to Address Misconceptions:

- Misconception: A frustum is the same as a cone. Clarification: No, a frustum is a cone with the top sliced off, creating two circular bases.
- Misconception: The formula is the same as the cone formula. Clarification: No, the frustum formula includes both radii: r squared + R squared + R r.
- Misconception: The height is the slant height. Clarification: No, the height is the perpendicular distance between the two bases.
- Misconception: I can use the diameter instead of the radius. Clarification: No, the formula uses the radius. If you have the diameter, divide by 2 to get the radius.

Phase 3: Practice and Application (10 minutes)

Worked Example:

A frustum of a cone has a top radius of 4 cm, a bottom radius of 8 cm, and a height of 10 cm.

a. Find the slant height of the frustum.

b. Find the volume of the frustum.

Solution:

a. Using the Pythagoras theorem, the slant height ℓ is given by:

ℓ = square root of (8 squared + 10 squared)

= square root of (64 + 100)

= square root of 164

= 12.80 cm

Slant height ℓ = 12.80 cm

b. Finding the volume:

V = (1 / 3) times π times h times (r squared + R squared + R r)

V = (1 / 3) times (22 / 7) times 10 cm times (4 squared + 8 squared + (8 times 4)) cm

= (22 / 21) times 10 cm times (16 cm + 64 cm + 32 cm)

= (220 / 21) times 112 cm squared

= 1173.33 cm cubed

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A square pyramid is cut into a frustum, where the original height was 18 cm, and the truncated top part has a height of 6 cm. The base side length is 12 cm, and the top side length is 6 cm. Find the volume of the frustum.

2. A frustum of a cone has a volume of 900 cm cubed, a base radius of 10 cm, and a top radius of 6 cm. Find its height.

3. An improvised jerrycan shaped like a frustum of a cone with a top radius of 20 cm, a bottom radius of 15 cm, and a height of 30 cm. How many liters of water can the bucket hold?

Answer Key:

1. For square pyramid frustum: V = (1 / 3) times h times (a squared + A squared + a A), where h = 12 cm, a = 6 cm, A = 12 cm. V = (1 / 3) times 12 times (36 + 144 + 72) = 1008 cm cubed.

2. V = (1 / 3) times π times h times (r squared + R squared + R r). 900 = (1 / 3) times 3.14 times h times (36 + 100 + 60). Solve for h: h = 4.37 cm.

3. V = (1 / 3) times π times 30 times (15 squared + 20 squared + 15 times 20) = 24,347 cm cubed = 24.35 liters.

Differentiation Strategies

For Struggling Learners:

- Provide pre-measured frustum models with labeled dimensions.
- Use frustums with simple dimensions for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators for calculations.
- Break down the formula into steps: Calculate r squared, R squared, R r, add them, multiply by h, multiply by π, divide by 3.

For On-Level Learners:

- Encourage students to verify their formula with water measurements.
- Ask students to explain the relationship between frustums and cones.
- Provide mixed practice with different dimensions.
- Challenge students to find the volume when only the diameters and height are given.

For Advanced Learners:

- Challenge students to derive the formula using the difference between two cones.
- Explore real-world applications: buckets, lampshades, flowerpots, jerrycans.
- Investigate the relationship between radii, height, and volume.
- Solve optimization problems: Given a fixed volume, what dimensions minimize the surface area?
- Apply the concept to frustums of pyramids with square or rectangular bases.

Extension Activity

Real-World Application: Designing a Bucket or Flowerpot

Work in groups

Situation: Design a frustum-shaped bucket or flowerpot for a specific purpose (e.g., water storage, planting flowers, collecting rainwater).

Tasks:

13. Choose a real-world purpose for your frustum.
14. Decide on the dimensions: top radius, bottom radius, and height.
15. Calculate the volume of your frustum using the formula.
16. If the bucket needs to hold 10 liters of water, what dimensions would work?
17. Compare different designs: wide top vs. narrow top, tall vs. short.
18. Present your findings with diagrams, measurements, and calculations.

Key Takeaway:

Students should understand how the volume of frustums is used in real-world contexts such as water storage (buckets, jerrycans), gardening (flowerpots), lighting (lampshades), and construction (truncated structures).

Teacher Reflection Prompts

- Did students successfully measure frustum-shaped containers?
- Were students able to calculate volume using the formula?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the relationship between frustums and cones?
- What adjustments would improve this lesson for future classes?