CBC Grade 10 Mathematics Lesson Plan

Representing Vectors Geometrically

Strand	Measurement and Geometry
Sub-Strand	Vectors 1
Specific Learning Outcome	Represent vectors geometrically using arrows, determine magnitude using distance formula, and add vectors in different situations
Key Inquiry Questions	How do we represent vectors geometrically and calculate their magnitudes?
Learning Resources	CBC Grade 10 textbooks, graph paper, rulers, pencils, calculators
Lesson Duration	40 minutes

Lesson Structure Overview

Phase	Activity	Duration
Phase 1	Problem-Solving and Discovery (Anchor Activity)	15 minutes
Phase 2	Structured Instruction (Key Takeaways)	10 minutes
Phase 3	Practice and Application (Worked Examples)	15 minutes
Phase 4	Assessment (Exit Ticket)	5 minutes

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Drawing and Measuring Vectors
Work in Groups (2-3 students)

Materials:

- • Ruler
- • Graph paper
- • Pencil
- • Calculator

Tasks:

1. (a) Draw vector AB from A(0,0) to B(4,2). Measure its magnitude using the distance formula.

2. (b) Draw another vector BC from B(4,2) to C(6,5). Add AB and BC.
3. (c) Draw two vectors from the origin: a = (3,1) and b = (2,4). Construct the parallelogram to determine a + b.
4. (d) Compare with other groups and discuss the results.

Teacher Guidance for Anchor Activity

This anchor activity introduces geometric representation of vectors through hands-on drawing and measurement. Students discover that vectors are represented by arrows, with length indicating magnitude and direction shown by the arrowhead. They also explore vector addition graphically.

Facilitation Strategy:

- • Organize students into groups of 2-3
- • Distribute graph paper, rulers, pencils, and calculators
- • Guide students to draw vector AB as an arrow from (0,0) to (4,2)
- • Help students apply distance formula: $|AB| = \sqrt{[(4-0)^2 + (2-0)^2]} = \sqrt{20} \approx 4.47$
- • For task (b), ensure BC starts where AB ends, then guide tip-to-tail addition
- • For task (c), demonstrate parallelogram method: draw both vectors from origin, complete parallelogram, diagonal is resultant
- • Ask probing questions: "What does the arrow length represent?" "Why do we use arrows?" "How is magnitude calculated?"
- • Use student observations to introduce formal notation and terminology

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

After students have explored through the anchor activity, formalize their discoveries with these key concepts:

1. Geometric Representation of Vectors

Vectors are typically represented using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow shows the direction of the vector.

We can represent a vector by a line drawn between two points. The direction is indicated by an arrowhead, while magnitude is determined by the length.

2. Vector Notation

A vector PQ can be denoted as:

- • PQ with arrow above: PQ→
- • PQ with tilde: PQ~
- • Single lowercase letter: a or a~

The magnitude of vector PQ is represented as |PQ|.

We refer to P as the initial point and Q as the terminal point.

3. Calculating Magnitude Using Distance Formula

For a vector from point $P(x_1, y_1)$ to point $Q(x_2, y_2)$:

Step 1: Determine the components

$PQ = (x_2 - x_1, y_2 - y_1)$

Step 2: Use the magnitude formula

$|PQ| = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

This is the distance formula, which gives the length of the vector.

4. Adding Vectors Geometrically

Two methods for geometric vector addition:

Tip-to-Tail Method: Place the tail of the second vector at the tip of the first. The resultant goes from the starting tail to the final tip.

Parallelogram Method: Draw both vectors from the same starting point. Complete the parallelogram. The diagonal from the starting point is the resultant.

5. Algebraic Addition of Vectors

When vectors are in coordinate form, add corresponding components:

If $a = (x_1, y_1)$ and $b = (x_2, y_2)$, then:

$a + b = (x_1 + x_2, y_1 + y_2)$

For subtraction:

$a - b = (x_1 - x_2, y_1 - y_2)$

6. Real-Life Applications

Navigation: Maps use arrows to show direction and distance, representing displacement vectors.

Physics: Force vectors are drawn as arrows, with length representing force magnitude.

Engineering: Structural diagrams use vector arrows to show loads and forces on buildings.

Scaffolding Strategies

Address common misconceptions revealed during the anchor activity:

- • Clarify that arrow length represents magnitude (not just any line)
- • Emphasize that arrowhead shows direction
- • Show that magnitude is always positive (it's a length)
- • Connect distance formula to Pythagorean theorem
- • Demonstrate both tip-to-tail and parallelogram methods produce same result

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Finding Magnitude

Find the magnitude of vector PQ where P(1,2) and Q(5,5).

Solution:

Step 1: Determine the components

PQ = (5-1, 5-2) = (4, 3)

Step 2: Use the magnitude formula

$|PQ| = \sqrt{(4^2 + 3^2)} = \sqrt{(16 + 9)} = \sqrt{25} = 5$

The magnitude is 5 units.

Example 2: Adding Vectors

Add vectors a = (2,3) and b = (4,1).

Solution:

Add corresponding components:

a + b = (2+4, 3+1) = (6, 4)

Therefore, the resultant vector is (6,4).

Example 3: Subtracting Vectors
Determine a - b if a = (5,2) and b = (1,4).

Solution:

Subtract corresponding components:

a - b = (5-1, 2-4) = (4, -2)

Therefore, the resulting vector is (4,-2).

Example 4: Sequential Vector Addition
Given A(0,0), B(2,1), and C(5,4), determine AB + BC.

Solution:

AB = (2-0, 1-0) = (2, 1)

BC = (5-2, 4-1) = (3, 3)

AB + BC = (2,1) + (3,3) = (5, 4)

Therefore, AB + BC = (5,4), which equals AC (from A to C).

Individual Practice (Students work independently)
Provide students with similar problems to solve:

5. 1. Find the magnitude of vector (3,4).
6. 2. Add vectors (1,2) and (3,5).
7. 3. If a = (6,-2) and b = (4,3), find 2a.
8. 4. Draw vector from origin to (3,2) on graph paper and measure its length.

Phase 4: Assessment - Exit Ticket (5 minutes)
Students complete individually to demonstrate understanding:

Question 1: Draw a vector from point A(1,1) to point B(4,5) on graph paper. Calculate its magnitude using the distance formula.

Question 2: Given vectors p = (2,3) and q = (5,1), find p + q.

Question 3: Explain why we use arrows to represent vectors. What do the length and direction of the arrow represent?

Exit Ticket Answer Key

Question 1:

Components: AB = (4-1, 5-1) = (3, 4)

Magnitude: $|AB| = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$

The magnitude is 5 units.

Question 2:

p + q = (2+5, 3+1) = (7, 4)

The resultant vector is (7,4).

Question 3:

We use arrows to represent vectors because vectors have both magnitude and direction. The length of the arrow represents the magnitude (how much), and the direction of the arrowhead shows the direction (which way). This visual representation makes it easy to understand and work with vectors in navigation, physics, and engineering.

Differentiation Strategies

For Struggling Learners:
- • Provide pre-drawn axes and coordinate grids
- • Use color coding for different vectors
- • Provide step-by-step templates for distance formula
- • Allow calculators for all magnitude calculations
- • Start with vectors along axes before diagonal vectors
- • Use physical arrows or string to demonstrate concepts

For Advanced Learners:
- • Introduce three-dimensional vectors
- • Explore unit vectors and vector decomposition
- • Challenge with finding vectors given magnitude and direction
- • Connect to trigonometry (angle calculations)
- • Apply to physics problems (velocity, force vectors)
- • Introduce vector multiplication (dot product)

Extension Activity

Vector Art Project
Objective: Create geometric art using vectors.

Activity Description:

9. 1. Design a simple shape or pattern using vectors on graph paper.
10. 2. Draw at least 5 vectors with different magnitudes and directions.
11. 3. Label each vector with its coordinates and calculate its magnitude.
12. 4. Color code vectors by magnitude range (e.g., 0-3 units = blue, 3-6 units = green, 6+ units = red).
13. 5. Present your vector art to the class, explaining the vectors used.

Navigation Challenge
Students can:

- • Plan a multi-step journey using vector arrows on a map
- • Calculate total displacement using vector addition
- • Compare actual distance traveled vs. displacement magnitude
- • Present navigation plan with vector diagrams

Post-Lesson Reflection for Teachers
- • Did students successfully represent vectors geometrically using arrows?
- • Were students able to calculate magnitudes using the distance formula?
- • What misconceptions emerged about vector direction and magnitude?
- • How engaged were students with the hands-on drawing activity?
- • Did students understand both tip-to-tail and parallelogram methods?
- • What adjustments are needed for future lessons on this topic?