Grade 10 Mathematics Lesson Plan

Tangents of Acute Angles

Strand:	**Measurement and Geometry**
Sub-Strand:	Trigonometry 1
Specific Learning Outcome:	Relate the sine, cosine and tangent of acute angles
Duration:	40 minutes
Key Inquiry Questions:	What is trigonometry? How do we use trigonometry in real-life situations?
Learning Resources:	CBC Grade 10 textbooks, rulers, pencils, protractors, graph paper

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering tangent ratios through similar triangles
Structured Instruction	10 minutes	Formalizing the tangent ratio definition and notation
Practice and Application	10 minutes	Worked examples and guided practice
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Tangent Ratios

Students work in pairs to draw a diagram with similar triangles, measure side lengths, calculate ratios, and discover that the ratio of vertical distance to horizontal distance remains constant for a given angle.

Materials Required:

- A piece of paper
- A ruler
- A pencil
- A protractor

Instructions for Students:

1. Using a piece of paper, a ruler, and a pencil, carefully draw the diagram shown (Figure 2.4.1 from textbook).
2. Measure and record the lengths of OB, BQ, OC, CR, OA, and AP using a ruler.
3. Identify whether triangles OPA, OQB, and ORC are similar. If they are similar, compare the ratios of their corresponding sides.
4. Calculate the following ratios: PA/OA, QB/OB, RC/OC
5. Observe the ratios from the previous step. What do you notice about the relationship between them?
6. Considering the parallel lines BQ, AP and RC, examine the relationship between the vertical and horizontal distances. What do you notice about their ratios?
7. Use a protractor to measure the angle marked x° in the diagram.
8. Share your observations and conclusions with your classmates.

Recording Table for Student Observations:

Measurement	Value	Notes
Lengths: OB, BQ, OC, CR, OA, AP	___	___
Ratio PA/OA	___	___
Ratio QB/OB	___	___
Ratio RC/OC	___	___

Teacher Role During Discovery:

- Circulate among pairs, ensuring students draw the diagram accurately with parallel lines.
- Ask probing questions: "Are the three triangles similar?" "What do you notice about the ratios?" "What does this tell you about the relationship between the angle and the ratio?"
- For struggling pairs: "Let us focus on triangle OPA first. Measure PA (vertical) and OA (horizontal). Now calculate PA divided by OA. Do the same for triangle OQB."
- For early finishers: "What if you measured a different angle? Would the ratio change? Try drawing another set of parallel lines at a different angle."
- Guide students to articulate: "The ratio of vertical to horizontal distance is the same for all similar triangles with the same angle. This ratio is called the tangent of the angle."
- Identify 2-3 pairs with clear results to share with the class.

Discovery Table: Linking Observations to Mathematical Significance

Student Observation	Mathematical Significance

The three triangles OPA, OQB, and ORC are similar	Similar triangles have equal ratios of corresponding sides
The ratios PA/OA, QB/OB, and RC/OC are all equal (1.5)	The ratio is constant for a given angle, regardless of triangle size
The ratio of vertical distance to horizontal distance is the same	This constant ratio defines the tangent of the angle
The angle x° is the same in all three triangles	The tangent depends only on the angle, not the triangle size
When we measure angle x°, we can associate it with the ratio 1.5	We write this as tan x° = 1.5

Phase 2: Structured Instruction (10 minutes)

Connecting Student Discoveries to Formal Concepts

After students have completed the anchor activity and shared their findings, the teacher formalizes the concept of tangent.

Key Takeaways:

- When given the triangle from the activity, the triangles OPA, OQB, and ORC are similar. This means that the ratios of their corresponding sides are equal: PA/OA = QB/OB = RC/OC = 15/10 = 1.5
- For any line parallel to BQ, the ratio of vertical distance to horizontal distance remains the same in each triangle. In this case, the ratio is 1.5.
- This constant ratio, Vertical distance / Horizontal distance, is called the tangent of angle VOT. Therefore, the tangent of x° is 1.5, which can be written as: tan x° = 1.5
- The tangent of an angle depends only on the size of the angle, not on the triangle size.

Formal Definition of Tangent:

The diagram shows a right-angled triangle ABC, where ∠ABC = θ.

- The side AC is the vertical side, which is opposite to angle θ.
- The side AB is the horizontal side, which is adjacent to angle θ.
- The side BC is the hypotenuse, which is the longest side of the right-angled triangle.

In this case, the tangent of angle θ is defined as the ratio of the opposite side to the adjacent side:

$\tan \theta = \text{Opposite side} / \text{Adjacent side} = AC / AB$

Scaffolding Strategies to Address Misconceptions:

- Misconception: "Tangent is just any ratio in a triangle." → Clarification: "Tangent is specifically the ratio of the opposite side to the adjacent side in a right-angled triangle."
- Misconception: "The tangent changes if I make the triangle bigger." → Clarification: "No, the tangent depends only on the angle. Similar triangles have the same tangent value."
- Misconception: "I can use any two sides." → Clarification: "You must identify which side is opposite the angle and which is adjacent. The hypotenuse is never used in the tangent ratio."

Phase 3: Practice and Application (10 minutes)

Worked Examples:

Example 1: Finding Tangent from Given Measurements (Textbook Example 2.4.5)

Find the tangent of the indicated angle using the given measurements.

Given: Opposite side = 3 cm, Adjacent side = 4 cm

Solution:

- $\tan \theta$ = Opposite / Adjacent
- $\tan \theta = 3$ cm / 4 cm
- $\tan \theta = 3/4$
- $\tan \theta = 0.75$

Therefore, $\tan \theta = 0.75$

Example 2: Finding Tangent Using Pythagorean Theorem (Textbook Example 2.4.6)

Find the tangent in the indicated angle below.

Given: Hypotenuse = 5 cm, Base = 3 cm

Solution:

Step 1: Calculate the perpendicular height using Pythagorean theorem.

- $H^2 = b^2 + h^2$
- $h^2 = H^2 - b^2 = 5^2 - 3^2 = 25 - 9 = 16$
- $h = 4$ cm

Step 2: Finding tan θ:

Opposite side = 3 cm, Adjacent side = 4 cm

tan θ = 3/4 = 0.75

Step 3: Finding tan α:

Opposite side = 4 cm, Adjacent side = 3 cm

tan α = 4/3 = 1.333

Example 3: Real-World Application - Flag Pole Problem

The inclination of the observer line of sight to the top of a 10 m high flag pole, positioned 15 m away, can be determined using a scale drawing.

Question: Express tan α in terms of the lengths of the sides of the triangle.

Solution:

In the right-angled triangle formed:

- Opposite side (height of flag pole) = 10 m
- Adjacent side (distance from observer) = 15 m
- tan α = Opposite / Adjacent
- tan α = 10 / 15
- tan α = 2/3
- tan $\alpha \approx 0.667$

The angle α is called the angle of elevation.

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually on a piece of paper.

1. Define the tangent of an angle in a right-angled triangle.

2. In a right-angled triangle, the opposite side is 6 cm and the adjacent side is 8 cm. Find tan θ.

3. If tan α = 1.5, and the adjacent side is 10 cm, what is the length of the opposite side?

Answer Key:

1. The tangent of an angle θ in a right-angled triangle is the ratio of the opposite side to the adjacent side: tan θ = Opposite / Adjacent

2. tan θ = 6/8 = 3/4 = 0.75

3. tan α = Opposite / Adjacent → 1.5 = Opposite / 10 → Opposite = 1.5 × 10 = 15 cm

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn diagrams with labeled sides to reduce cognitive load.
- Use color-coding: opposite side in red, adjacent side in blue.
- Start with simple whole-number ratios (e.g., 3:4, 5:12) before moving to decimals.
- Provide a tangent ratio reference card with the formula and a labeled diagram.

For On-Level Learners:

- Encourage students to explain their reasoning when calculating tangent ratios.
- Provide mixed practice with different triangle orientations.
- Ask students to verify their answers using a calculator.

For Advanced Learners:

- Challenge students to find the angle when given the tangent value (inverse tangent).
- Explore the relationship between tangent and the other trigonometric ratios (sine and cosine).
- Investigate what happens to the tangent as the angle approaches 90°.
- Apply tangent to real-world problems involving angles of elevation and depression.

Extension Activity

Tables of Tangents Investigation

Students work in groups to explore tangent tables and learn how to read tangent values for different angles.

Materials: Printed Table of Tangents, 30 cm ruler, pencil, calculator

Tasks:

9. What is the tangent of an angle?
10. How do we use a Table of Tangents?
11. Use your Table of Tangents to find the following tangents: 42°, 35°, 90°, 42°47'
12. Discuss your findings with other groups in your class.

Key Takeaway:

Special tables have been prepared and can be used to obtain tangents of acute angles. The technique of reading tables of tangents is similar to that of reading tables of logarithms or square roots.

Note: In the tables of tangents, the angles are expressed in decimals and degrees or in degrees and minutes. One degree is equal to 60 minutes (60'). Thus: 30' = 0.50°, 54' = 0.9°, and 6' = 0.1°.

Teacher Reflection Prompts

- Did students successfully discover the constant ratio in the anchor activity?
- Were students able to identify opposite and adjacent sides correctly?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did the real-world examples help students see the relevance of tangent ratios?
- What adjustments would improve this lesson for future classes?