CBC Grade 10 Mathematics

Step-by-Step Presentation Script

Special Angles: 0, 45, and 90 Degrees

Pre-Class Preparation

Before students arrive, ensure the following materials and setup are ready:

- Materials Needed:
- • Rulers (one per group)
- • Pencils
- • Protractors (one per group)
- • Graph paper (several sheets per group)
- • Scientific calculators (for verification)
- • Chart paper for displaying key formulas
- • Pre-drawn triangle diagrams (for struggling learners)
- Classroom Setup:
- • Arrange desks for group work (groups of 3-4 students)
- • Prepare board space for drawing triangles and writing formulas
- • Have reference table of special angles ready to display
- • Print exit tickets (one per student)

Lesson Overview (40 Minutes)

Phase	Duration
Phase 1: Problem-Solving and Discovery	0-15 minutes
Phase 2: Structured Instruction	15-25 minutes
Phase 3: Practice and Application	25-37 minutes
Phase 4: Assessment (Exit Ticket)	37-40 minutes

Minute-by-Minute Presentation Guide

Minutes 0-2: Introduction and Engagement

[SAY] "Good morning, class! Today we are going to explore something fascinating in trigonometry—special angles. These are angles whose trigonometric ratios have exact values that we can calculate without using calculators or tables."

[ASK] "Who can tell me what trigonometry is? How do we use it in real life?"

[LISTEN] Allow 2-3 students to share. Acknowledge responses.

[SAY] "Excellent! Trigonometry helps us solve problems involving angles and distances. Architects use it to design roofs, engineers use it for bridges, and surveyors use it to measure land. Today, we'll discover the exact values for angles like 30°, 45°, 60°, and 90°."

[WRITE] On the board: "Special Angles: 0°, 45°, and 90°"

Minutes 2-17: Phase 1 - Anchor Activity (Discovery)

[DO] Organize students into groups of 3-4.

[SAY] "You will now work in groups to discover something amazing about special angles. Each group will receive rulers, protractors, and graph paper. Your task is to construct two special triangles and calculate their trigonometric ratios."

[WRITE] Display the anchor activity instructions on the board:

- Task 1: Drawing a 45-45-90 Triangle
- Task 2: Drawing a 30-60-90 Triangle
- Task 3: Measure and Calculate
- Task 4: Discussion

[SAY] "For Task 1, draw a square, then draw a diagonal to create two triangles. Label the angles as 45°, 45°, and 90°."

[SAY] "For Task 2, draw an equilateral triangle, then draw a line from one corner to the middle of the opposite side. This creates a 30-60-90 triangle."

[SAY] "Once you've drawn both triangles, measure the sides carefully and calculate sin, cos, and tan for the special angles. Record your findings in a table."

[DO] Distribute materials to each group.

[DO] Circulate among groups (Minutes 2-12). Observe their work and ask probing questions:

- • "What do you notice about the two sides of the 45-45-90 triangle?"
- • "How does the hypotenuse compare to the other sides?"
- • "What patterns do you see in the ratios?"

[TEACHING TIP] If groups struggle with construction, provide pre-drawn templates but encourage them to measure independently.

[DO] At minute 12, bring the class together for sharing.

[SAY] "Let's hear what you discovered. Group 1, what did you find for sin 45°?"

[LISTEN] Allow 2-3 groups to share their findings for 45°, 30°, and 60°.

[ASK] "Did anyone notice something special about sin 45° and cos 45°?"

[LISTEN] Students should notice they are equal.

[SAY] "Excellent observation! Now let's formalize what you've discovered."

Minutes 17-25: Phase 2 - Structured Instruction

[SAY] "You've done a wonderful job discovering patterns. Now let's connect your findings to the mathematical principles behind special angles."

[WRITE] Draw an isosceles right triangle on the board with sides labeled 1, 1, and hypotenuse.

[SAY] "This is a 45-45-90 triangle. Notice that two sides are equal—both have length 1. Let's find the hypotenuse using Pythagoras' theorem."

[WRITE] $PR^2 = 1^2 + 1^2 = 2$, so $PR = \sqrt{2}$

[SAY] "Now we can calculate the trigonometric ratios:"

[WRITE]

- $\sin 45° = \text{opposite/hypotenuse} = 1/\sqrt{2} = \sqrt{2}/2$
- $\cos 45° = \text{adjacent/hypotenuse} = 1/\sqrt{2} = \sqrt{2}/2$
- $\tan 45° = \text{opposite/adjacent} = 1/1 = 1$

[POINT] "Notice that sin 45° equals cos 45°. This makes sense because in an isosceles right triangle, the opposite and adjacent sides are equal!"

[SAY] "Now let's look at 30° and 60° angles, which come from an equilateral triangle."

[DRAW] An equilateral triangle ABC with side length 2, and perpendicular bisector AD.

[SAY] "When we draw a perpendicular from one vertex to the opposite side, we create two 30-60-90 triangles."

[WRITE] $AD^2 = 2^2 - 1^2 = 4 - 1 = 3$, so $AD = \sqrt{3}$

[SAY] "Using this triangle, we can find:"

[WRITE]

- $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, $\tan 30° = 1/\sqrt{3}$
- $\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$, $\tan 60° = \sqrt{3}$

[POINT] "Notice something interesting: $\sin 30° = \cos 60°$ and $\cos 30° = \sin 60°$. The sine and cosine are swapped!"

[SAY] "Finally, let's consider the extreme angles: 0° and 90°."

[WRITE]

- $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$
- $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

[ASK] "Why do you think $\tan 90°$ is undefined?"

[LISTEN] Guide students to understand that tan = sin/cos, and dividing by zero is undefined.

[TEACHING TIP] Create a reference table on chart paper with all special angle values for students to refer to during practice.

Minutes 25-37: Phase 3 - Practice and Application

[SAY] "Now that we understand the exact values, let's apply them to solve problems without using calculators."

[EXAMPLE] Example 1: Simplifying Expressions

[WRITE] "Simplify: $\sin 30° \cos 45°$"

[SAY] "Let's substitute the exact values we learned:"

[WRITE] $\sin 30° \cos 45° = (1/2) \times (1/\sqrt{2}) = 1/(2\sqrt{2})$

[SAY] "We can rationalize this: $1/(2\sqrt{2}) \times \sqrt{2}/\sqrt{2} = \sqrt{2}/4$"

[WRITE] Final answer: $\sqrt{2}/4$

[EXAMPLE] Example 2: More Complex Expression

[WRITE] "Simplify: $8 \cos 45° \sin 45°$"

[SAY] "Let's work through this step by step:"

[WRITE] $8 \cos 45° \sin 45° = 8 \times (1/\sqrt{2}) \times (1/\sqrt{2}) = 8 \times 1/2 = 4$

[EXAMPLE] Example 3: Real-World Application

[SAY] "Here's a practical problem: The angle made by the arms of an upright pair of dividers and the horizontal is 45°. The vertical distance from the horizontal to the vertex is 15 cm. Find the horizontal distance between the tips of the arms."

[DRAW] Sketch the dividers forming a triangle.

[SAY] "Since the angle is 45° and we have an isosceles triangle, the horizontal distance on each side equals the vertical distance."

[WRITE] Each side = 15 cm, so total horizontal distance = 15 + 15 = 30 cm

[SAY] "For part (b), to find the length of the arms, we use Pythagoras:"

[WRITE] $\text{Length}^2 = 15^2 + 15^2 = 450$, so Length = $\sqrt{450} = 15\sqrt{2}$ cm

[DO] Give students 5 minutes (minutes 32-37) to work on individual practice problems:

1. 1. Find exact values: sin 0°, cos 0°, tan 0°
2. 2. Find exact values: sin 45°, cos 45°, tan 45°
3. 3. Simplify: sin 45° + cos 45°

[DO] Circulate to check understanding and provide support.

[TEACHING TIP] Encourage students to draw triangles if they forget the values. The geometry helps them remember!

Minutes 37-40: Phase 4 - Assessment (Exit Ticket)

[SAY] "Excellent work today! To check your understanding, you'll complete a short exit ticket individually."

[DO] Distribute exit tickets to each student.

[SAY] "You have 3 minutes to complete these questions. Remember to show your work and use exact values, not decimals."

[WRITE] Display exit ticket questions on the board:

4. 1. Without using a calculator, find: sin 60° + cos 30°
5. 2. Given sin θ = 1/2, find angle θ ($0° \leq \theta \leq 90°$)
6. 3. The angle is 45°, vertical distance is 15 cm. Find horizontal distance and arm length.

[DO] Students work silently on exit tickets (minutes 37-40).

[DO] Collect exit tickets as students finish.

[SAY] "Thank you for your excellent work today! Tomorrow we'll explore how to use these special angles in more complex problems. For homework, practice the exercises on page [X] of your textbook."

Teaching Tips and Strategies

- Emphasis Points:
- • Stress that these are exact values, not approximations
- • Connect geometric constructions to algebraic ratios
- • Use visual diagrams throughout—trigonometry is inherently visual
- • Encourage students to draw triangles when they forget values
- • Highlight the swap pattern between 30° and 60°
- Differentiation in Action:
- • For struggling learners: Provide pre-drawn triangles and reference tables
- • For advanced learners: Challenge them with angle addition formulas (e.g., sin 75°)
- • Use peer teaching—pair strong students with those who need support
- Common Student Errors to Watch For:
- • Confusing sin 30° with sin 60° (emphasize the swap pattern)

- • Writing decimal approximations instead of exact values
- • Forgetting to rationalize denominators (e.g., leaving $1/\sqrt{2}$ instead of $\sqrt{2}/2$)
- • Thinking $\tan 90° = 0$ (remind them it's undefined)
- • Mixing up opposite and adjacent sides in triangles
- Engagement Strategies:
- • Use real-world contexts (architecture, engineering)
- • Create memory aids: "30° is small, so $\sin 30° = 1/2$ is small"
- • Challenge students to find special angles in the classroom (corners, roof angles)
- • Use hand gestures to show angle sizes

Assessment Guidance

Exit Ticket Evaluation Criteria:

- • Correct use of exact values (not decimal approximations)
- • Proper mathematical notation and working
- • Ability to apply special angles to solve problems
- • Understanding of when $\tan 90°$ is undefined

Mastery Indicators:

- • Student can recall exact values for 30°, 45°, 60° without hesitation
- • Student can derive values using triangle geometry
- • Student can simplify expressions involving special angles
- • Student can apply special angles to real-world problems

Follow-Up for Students Who Struggle:

- • Provide additional practice with visual aids
- • Create flashcards for memorization
- • Schedule small group intervention
- • Use online interactive tools for extra practice

Post-Lesson Reflection Questions

After teaching this lesson, reflect on the following:

- • Did students successfully construct the special triangles during the anchor activity?
- • Were students able to discover the patterns before formal instruction?
- • How well did students transition from geometric construction to algebraic calculation?
- • What misconceptions emerged, and how were they addressed?
- • Did the real-world examples resonate with students?
- • What percentage of students demonstrated mastery on the exit ticket?
- • What adjustments would improve this lesson for next time?