CBC Grade 10 Mathematics Lesson Plan

Special Angles: 0, 45, and 90 Degrees

Strand	Measurement and Geometry
Sub-Strand	Trigonometric Ratios of Special Angles
Specific Learning Outcome	Determine trigonometric ratios of special angles: 30, 45, 60, and 90 degrees using triangles
Key Inquiry Questions	What is trigonometry? How do we use trigonometry in real-life situations?
Learning Resources	CBC Grade 10 textbooks, rulers, pencils, protractors, graph paper, scientific calculators
Lesson Duration	40 minutes

Lesson Structure Overview

Phase	Activity	Duration
Phase 1	Problem-Solving and Discovery (Anchor Activity)	15 minutes
Phase 2	Structured Instruction (Key Takeaways)	10 minutes
Phase 3	Practice and Application (Worked Examples)	15 minutes
Phase 4	Assessment (Exit Ticket)	5 minutes

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Special Angles

Work in groups

What you require: Ruler, pencil, protractor, graph paper.

Task 1: Drawing a 45-45-90 Triangle

1. (i) Draw a square.
2. (ii) Draw a diagonal to make two triangles.
3. (iii) Label angles (45°, 45°, 90°).

Task 2: Draw 30-60-90 Triangle

4. (i) Draw an equilateral triangle.
5. (ii) Draw a line from a corner to the middle of the opposite side.
6. (iii) Label angles (30°, 60°, 90°).

Task 3: Measure and Calculate

- • Measure sides of each triangle.
- • Calculate sin, cos, tan for 30°, 45°, 60° angles.
- • Record your results in the table below.

Task 4: Discussion

Discuss your result with others. What do you notice for the Special Angles (30°, 45°, 60°)?

Teacher Guidance for Anchor Activity

This anchor activity activates prior knowledge of triangles, angles, and basic trigonometric ratios. Students work collaboratively to construct special triangles and discover patterns in their trigonometric ratios.

- Facilitation Strategy:
- • Circulate among groups to ensure accurate constructions
- • Ask probing questions: "What do you notice about the sides?" "How do the ratios compare?"
- • Encourage students to look for patterns across different angles
- • Use student discoveries as a bridge to formal instruction

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

After students have explored through the anchor activity, formalize their discoveries with these key concepts:

1. Special Angle 45° (Isosceles Right Triangle)

An isosceles triangle is a triangle whose two sides and base angles are equal.

For a 45-45-90 triangle with equal sides of length 1:

- • Using Pythagoras: $PR^2 = 1^2 + 1^2 = 2$, so $PR = \sqrt{2}$
- • $\sin 45° = 1/\sqrt{2} = \sqrt{2}/2$
- • $\cos 45° = 1/\sqrt{2} = \sqrt{2}/2$
- • $\tan 45° = 1$

2. Special Angles 30° and 60° (Equilateral Triangle)

The figure shows an equilateral triangle ABC with side length 2. AD is the perpendicular bisector of BC.

Notice that:

- • Length AD is given by: $AD^2 = 2^2 - 1^2 = 4 - 1 = 3$, so $AD = \sqrt{3}$

Therefore:

- • $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, $\tan 30° = 1/\sqrt{3}$
- • $\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$, $\tan 60° = \sqrt{3}$

3. Special Angles 0° and 90°

- • $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$
- • $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

4. Key Observations

- • Sin and Cos are swapped between 30° and 60°
- • tan 30° is small ($1/\sqrt{3} \approx 0.577$) while tan 60° is large ($\sqrt{3} \approx 1.732$)
- • $\sin\theta \times \sin\theta$ can be written as $\sin^2\theta$
- • For 45°, sine and cosine are equal

Scaffolding Strategies

Address common misconceptions revealed during the anchor activity:

- • If students struggle with Pythagoras, review the theorem with concrete examples
- • Emphasize that these exact values are derived from geometry, not approximations
- • Connect the ratios to the actual triangle measurements students made
- • Use visual diagrams to reinforce the relationship between angles and sides

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Simplifying Trigonometric Expressions

Simplify the following without using tables or calculators:

(a) $\sin 30° \cos 45°$

Solution:

$\sin 30° \cos 45° = (1/2) \times (1/\sqrt{2}) = 1/(2\sqrt{2}) = \sqrt{2}/4$

(b) $8 \cos 45° \sin 45°$

Solution:

$8 \cos 45° \sin 45° = (8 \times 1/\sqrt{2}) \times (1/\sqrt{2}) = 8/(\sqrt{2} \times \sqrt{2}) = 8/2 = 4$

(c) $\sin 60° \cos 45° + \sin 30° \tan 45°$

Solution:

$\sin 60° \cos 45° + \sin 30° \tan 45° = (\sqrt{3}/2 \times 1/\sqrt{2}) + (1/2 \times 1)$

$= \sqrt{3}/(2\sqrt{2}) + 1/2 = \sqrt{3}/(2\sqrt{2}) + \sqrt{2}/\sqrt{2} \times 1/2 = (\sqrt{3} + \sqrt{2})/(2\sqrt{2})$

Example 2: Real-Life Application (Dividers Problem)

The angle made by the arms of an upright pair of dividers and the horizontal is 45°. The vertical distance from the horizontal to the vertex is 15 cm. Find without using tables:

(a) The horizontal distance between the tips of the arms.

Solution:

Since the angle is 45°, tan 45° = 1

Let horizontal distance = x

tan 45° = vertical distance / (horizontal distance/2)

1 = 15 / (x/2)

x/2 = 15

x = 30 cm

The horizontal distance is 30 cm.

(b) The length of the arms.

Solution:

Using Pythagoras theorem:

$Length^2 = 15^2 + 15^2 = 225 + 225 = 450$

Length = √450 = 15√2 cm

The length of each arm is 15√2 cm.

Individual Practice (Students work independently)

Provide students with similar problems to solve:

7. 1. Find exact values: sin 0°, cos 0°, tan 0°
8. 2. Find exact values: sin 45°, cos 45°, tan 45°
9. 3. Find exact values: sin 90°, cos 90°, tan 90°
10. 4. Simplify: sin 45° + cos 45°
11. 5. Given θ = 45°, calculate: $\tan^2\theta - \sin^2\theta$

Phase 4: Assessment - Exit Ticket (5 minutes)

Students complete individually to demonstrate understanding:

Question 1: Without using a calculator, find the exact value of sin 60° + cos 30°.

Question 2: The angle made by the arms of an upright pair of dividers and the horizontal is 45°. The vertical distance from the horizontal to the vertex is 15 cm. Find without using tables:

- (a) The horizontal distance between the tips of the arms.
- (b) The length of the arms.

Question 3: Given: sin θ = 1/2. Find the angle θ, where 0° ≤ θ ≤ 90°.

Question 4: Using the triangle with sides in ratio 1:1:√2, calculate the value of:

- (a) sin 30°
- (b) cos 60°
- (c) tan 30°

Exit Ticket Answer Key

Question 1: sin 60° + cos 30° = √3/2 + √3/2 = √3

Question 2: (a) 30 cm, (b) 15√2 cm

Question 3: θ = 30°

Question 4: (a) 1/2, (b) 1/2, (c) 1/√3

Differentiation Strategies

For Struggling Learners:

- • Provide pre-drawn triangles with measurements labeled
- • Use a reference table of special angle values for initial practice
- • Allow use of calculators to verify exact values
- • Pair with stronger students during group work
- • Focus on memorizing the most common values (30°, 45°, 60°) first
- • Provide step-by-step worked examples with annotations

For Advanced Learners:

- • Challenge them to derive values for 15° and 75° using angle addition formulas
- • Explore the unit circle representation of special angles
- • Investigate why tan 90° is undefined
- • Create their own real-world problems involving special angles
- • Explore trigonometric identities using special angles
- • Investigate the relationship between special angles and regular polygons

Extension Activity

Special Angles Memory Challenge

Objective: Develop fluency with special angle values through pattern recognition.

Activity Description:

12. 1. Create a set of cards with angles (0°, 30°, 45°, 60°, 90°) and another set with their trigonometric values.
13. 2. Students work in pairs to match angles with their correct sin, cos, and tan values.
14. 3. Time the activity and challenge students to improve their speed.
15. 4. Extend by including expressions like $\sin^2 45°$ or $2\cos 30°$.

Real-World Investigation: Architecture and Special Angles

Students research and identify real-world structures that use 30°, 45°, or 60° angles:

- • Roof pitches (often 30° or 45°)
- • Staircase angles (typically around 30-45°)
- • Bridge supports and trusses
- • Ramps for accessibility (often use specific angles)

Students calculate dimensions using special angle trigonometry and present their findings.

Post-Lesson Reflection for Teachers

- • Did students successfully discover the patterns in special angle ratios during the anchor activity?
- • Were students able to apply exact values without calculators?
- • What misconceptions emerged during the lesson?
- • How well did students connect geometric constructions to trigonometric ratios?
- • What adjustments are needed for future lessons on this topic?