Grade 10 Mathematics Lesson Plan

Sines and Cosines of Complementary Angles

Strand:	**Measurements and Geometry**
Sub-Strand:	Trigonometry 1: Sines and Cosines of Complementary Angles
Specific Learning Outcome:	Relate sines and cosines of complementary angles
Key Inquiry Questions:	What is trigonometry? How do we use trigonometry in real-life situations?
Learning Resources:	CBC Grade 10 textbooks, Scientific calculators, Sine and cosine tables, Rulers, Pencils
Duration:	40 minutes
Class:	Grade 10

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity

Discovering the Complementary Angle Relationship

Work in pairs

What you require: A pencil, a ruler, a scientific calculator (for verification), working material and a printed sine and cosine table.

Task 1: Reading Values from Tables (5 minutes)

Read from the tables the values of the following pairs of angles:

- a) sin 40° and cos 50°
- b) cos 30° and sin 60°
- c) sin 70° and cos 20°
- d) sin 80° and cos 10°

Task 2: Pattern Recognition (3 minutes)

What do you notice about the results obtained in Task 1? Write down your observations.

Task 3: Class Discussion (5 minutes)

Discuss your work with other learners in your class:

- What pattern did you observe?
- What is the relationship between the two angles in each pair?

- What is the sum of the angles in each pair?

Teacher's Role:

- Circulate among pairs, ensuring students are reading tables correctly
- Ask guiding questions: 'What do you notice about 40° and 50°? What's their sum?'
- Encourage students to look for patterns across all four pairs
- Surface student thinking by asking pairs to share observations
- Bridge from discovery to formal definition of complementary angles

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

1. Definition of Complementary Angles

Complementary angles are two angles whose sum is 90° (or $\pi/2$ radians).

- Example: 40° and 50° are complementary because 40° + 50° = 90°
- Example: 30° and 60° are complementary because 30° + 60° = 90°

2. The Co-Function Identity

The sine and cosine functions of complementary angles have a special relationship:

- $\sin(90° - \theta) = \cos \theta$
- $\cos(90° - \theta) = \sin \theta$

3. General Rule for Complementary Angles

For any two complementary angles x and y where $x + y = 90°$:

- $\sin x = \cos y$
- $\cos x = \sin y$

This means: The sine of one angle equals the cosine of its complement, and vice versa.

4. Why 'Co-sine'?

The term 'cosine' literally means 'complement's sine' – it's the sine of the complementary angle!

5. Solving Strategy

When you see equations like:

- If $\sin A = \cos B$, then $A + B = 90°$
- If $\cos A = \sin B$, then $A + B = 90°$

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Finding Angles

Find acute angles θ and β if:

(a) $\cos 45° = \sin \alpha$

(b) $\cos \beta = \sin 5\beta$

(c) $\sin 2\alpha = \cos 30°$

Solution:

(a) $\cos 45° = \sin \alpha$

If $\cos 45° = \sin \alpha$, then the angles are complementary:

- $45° + \alpha = 90°$
- $\alpha = 90° - 45°$
- $\alpha = 45°$

(b) $\cos \beta = \sin 5\beta$

If $\cos \beta = \sin 5\beta$, then:

- $\beta + 5\beta = 90°$
- $6\beta = 90°$
- $\beta = 90°/6 = 15°$

(c) $\sin 2\alpha = \cos 30°$

If $\sin 2\alpha = \cos 30°$, then:

- $2\alpha + 30° = 90°$
- $2\alpha = 90° - 30° = 60°$
- $\alpha = 60°/2 = 30°$

Example 2: Complementary Angles with Ratios

Problem: A and B are complementary angles. If $A = \frac{1}{2}B$, find: (a) $\sin A$ (b) $\cos A$

Solution:

Since A and B are complementary: $A + B = 90°$

Given: $A = \frac{1}{2}B$, therefore $B = 2A$

Substitute into $A + B = 90°$:

- $A + 2A = 90°$
- $3A = 90°$
- $A = 30°$

Therefore:

- (a) $\sin A = \sin 30° = 0.5000$
- (b) $\cos A = \cos 30° = 0.8660$

Phase 4: Assessment (Exit Ticket)

1. If θ is an acute angle such that $\sin(\theta) = 3/5$, find $\cos(90° - \theta)$.

2. In a right-angled triangle, find $\sin \theta$ and $\cos \theta$, then verify the complementary relationship.

3. Given that $\cos(32°) = 0.848$, find the value of $\sin(58°)$ without using a calculator.

4. A ladder leans against a wall, making a 65° angle with the ground. Find the height at which the ladder touches the wall if the ladder is 10 m long.

Differentiation Strategies

For Struggling Learners:

- Provide pre-filled angle pair tables for pattern recognition
- Use right triangle diagrams to visualize complementary angles
- Start with simpler angles (30°, 60°, 45°)
- Provide formula cards with $\sin(90° - \theta) = \cos \theta$
- Allow extra time with calculators and tables

For Advanced Learners:

- Explore complementary angle relationships with tangent: $\tan(90° - \theta) = \cot \theta$
- Investigate supplementary angle relationships
- Solve complex equations involving multiple complementary pairs
- Research historical development of trigonometric identities
- Apply to real-world surveying or navigation problems

Extension Activity

Clinometer Investigation:

- Build a simple clinometer using a protractor and string
- Measure the angle of elevation to the top of a building or tree
- Calculate the complementary angle (angle from horizontal)
- Use both angles to find the height using trigonometry
- Verify that sin(angle of elevation) = cos(complementary angle)
- Present findings with diagrams and calculations