Grade 10 Mathematics Lesson Plan

Sines and Cosines of Acute Angles

Strand:	**Measurement and Geometry**
Sub-Strand:	Sine and Cosine of an Acute Angle
Specific Learning Outcome:	Relate the sine, cosine and tangent of acute angles
Duration:	40 minutes
Key Inquiry Questions:	What is trigonometry? How do we use trigonometry in real-life situations?
Learning Resources:	CBC Grade 10 textbooks, rulers, pencils, protractors, graph paper

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering sine and cosine ratios through similar triangles
Structured Instruction	10 minutes	Formalizing sine and cosine definitions and formulas
Practice and Application	10 minutes	Worked examples and guided practice
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering Sine and Cosine Ratios

Students work in groups to draw a diagram with similar triangles, measure side lengths, calculate ratios, and discover that two constant ratios emerge: the ratio of opposite to hypotenuse (sine) and the ratio of adjacent to hypotenuse (cosine).

Materials Required:

- A piece of paper
- A ruler
- A pencil
- A protractor

Instructions for Students:

1. The figure shows AP, BQ, and CR perpendicular to OV and angle TOV = θ (Figure 2.4.12 from textbook).
2. Copy the above figure in your writing materials.
3. Measure lengths OA, OP, AP, OQ, OB, BQ, OR, OC and CR.
4. Fill in the following: (i) AP/OP = ____, (ii) BQ/OQ = ____, (iii) CR/OR = ____
5. What do you notice about the ratios of (i...iii)?
6. Fill also the following: (i) OA/OP = ____, (ii) OB/OQ = ____, (iii) OC/OR = ____
7. What do you notice about these ratios above?
8. Discuss your findings with other groups in your class.

Recording Table for Student Observations:

Measurement	Value	Notes
Lengths: OA, OP, AP, OQ, OB, BQ, OR, OC, CR	___	___
Ratio AP/OP	___	___
Ratio BQ/OQ	___	___
Ratio CR/OR	___	___
Ratio OA/OP	___	___
Ratio OB/OQ	___	___

Teacher Role During Discovery:

- Circulate among groups, ensuring students draw the diagram accurately with perpendicular lines.
- Ask probing questions: "Are the three triangles similar?" "What do you notice about the first set of ratios?" "What about the second set of ratios?"
- For struggling groups: "Focus on triangle OPA first. Measure AP (opposite) and OP (hypotenuse). Now calculate AP divided by OP. Do the same for triangle OQB."
- For early finishers: "Can you predict what would happen if we drew more perpendicular lines at the same angle? Would the ratios stay the same?"
- Guide students to articulate: "The ratio of opposite to hypotenuse is constant. The ratio of adjacent to hypotenuse is also constant. These ratios depend only on the angle."
- Identify 2-3 groups with clear results to share with the class.

Discovery Table: Linking Observations to Mathematical Significance

Student Observation	Mathematical Significance

The ratios AP/OP, BQ/OQ, and CR/OR are all equal	This constant ratio is the sine of the angle θ
The ratios OA/OP, OB/OQ, and OC/OR are all equal	This constant ratio is the cosine of the angle θ
The first set of ratios uses opposite side and hypotenuse	sin θ = Opposite / Hypotenuse
The second set of ratios uses adjacent side and hypotenuse	cos θ = Adjacent / Hypotenuse
Both ratios are constant for a given angle	Sine and cosine depend only on the angle, not the triangle size

Phase 2: Structured Instruction (10 minutes)

Connecting Student Discoveries to Formal Concepts

After students have completed the anchor activity and shared their findings, the teacher formalizes the concepts of sine and cosine.

Key Takeaways:

- The ratios of (3) are the same and is expressed as: AP/OP = BQ/OQ = CR/OR. This constant value is obtained by taking the ratio of the side opposite to the angle θ to the hypotenuse side in each case. This ratio is called the sine of angle θ, which can be written as sin θ.
- The ratios of (5) are the same and is expressed as: OA/OP = OB/OQ = OC/OR. This constant value is obtained by taking the ratio of the side adjacent to the angle θ to the hypotenuse side in each case. This ratio is called the cosine of angle θ, which can be written as cos θ.

General Formulas for Trigonometric Ratios:

In general, given a right-angled triangle with opposite side, adjacent side and hypotenuse side:

Trigonometric Ratio	Formula
Tangent	tan θ = Opposite / Adjacent
Cosine	cos θ = Adjacent / Hypotenuse
Sine	sin θ = Opposite / Hypotenuse

The above formulas also apply to the trigonometric ratios for α.

Scaffolding Strategies to Address Misconceptions:

- Misconception: "Sine and cosine are the same thing." → Clarification: "No, sine uses opposite and hypotenuse, while cosine uses adjacent and hypotenuse."
- Misconception: "I can use any two sides for sine or cosine." → Clarification: "No, sine always uses opposite and hypotenuse. Cosine always uses adjacent and hypotenuse."
- Misconception: "The values change if I make the triangle bigger." → Clarification: "No, sine and cosine depend only on the angle. Similar triangles have the same sine and cosine values."
- Misconception: "Sine and cosine can be greater than 1." → Clarification: "No, since the hypotenuse is always the longest side, sine and cosine values range from 0 to 1."

Phase 3: Practice and Application (10 minutes)

Worked Examples:

Example 1: Finding Sine and Cosine (Textbook Example 2.4.13)

In the figure, MN = 5 cm, NO = 12 cm and angle MNO = 90°. Calculate:

a) sin θ

b) cos θ

Solution:

a) sin θ = opposite/hypotenuse = MN/MO = 5/MO

Recall: $MO^2 = 12^2 + 5^2 = 144 + 25 = 169$

MO = 13 cm

Thus, sin θ = 5/13 = 0.3846

b) cos θ = adjacent/hypotenuse = NO/MO = 12/13 = 0.9231

Example 2: Real-World Application - Ladder Problem (Textbook Example 2.4.14)

A ladder leans against a wall, forming a 70° angle with the ground. If the ladder is 5 meters long, how high does it reach on the wall?

Solution:

Using sin, since we need the opposite side:

- sin 70° = height/hypotenuse
- 0.9397 = height/5
- height = 5 × 0.9397
- height = 4.6985 m

The ladder reaches 4.6985 m up the wall.

Example 3: Finding Sine and Cosine from a Figure

In the figure given, find:

a) sin α

b) cos α

Solution:

a) sin α = opposite/hypotenuse = 3/5 = 0.6

b) cos α = adjacent/hypotenuse = 4/5 = 0.8

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually on a piece of paper.

1. In the figure given, find: a) sin α, b) cos α

2. A flagpole 12 meters tall casts a shadow of 8 meters on the ground. a) What is the angle of elevation of the sun? b) If the shadow increases to 10 meters, what will be the new angle of elevation?

3. An airplane takes off at an angle of 18° to the ground. After flying 500 meters, a) How high is the airplane above the ground? b) How far has it traveled horizontally from the starting point?

Answer Key:

1. a) sin α = 0.6, b) cos α = 0.8

2. a) The angle of elevation of the sun is 56.31°, b) The new angle of elevation is 50.19°

3. a) height = 154.51 m, b) horizontal distance = 475.53 m

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn diagrams with labeled sides (opposite, adjacent, hypotenuse).
- Use color-coding: opposite side in red, adjacent side in blue, hypotenuse in green.
- Start with simple whole-number ratios before moving to decimals.
- Provide a trigonometric ratios reference card with all three formulas.

For On-Level Learners:

- Encourage students to explain their reasoning when choosing which ratio to use.
- Provide mixed practice with different triangle orientations.
- Ask students to verify their answers using a calculator.

For Advanced Learners:

- Challenge students to find the angle when given the sine or cosine value (inverse functions).
- Explore the relationship between sine, cosine, and tangent: $\tan \theta = \sin \theta / \cos \theta$.
- Investigate complementary angles: $\sin \theta = \cos(90° - \theta)$.
- Apply sine and cosine to more complex real-world problems.

Extension Activity

Exploring the Relationship Between Sine, Cosine, and Tangent

Students work in groups to explore the relationship between the three trigonometric ratios.

Materials: Ruler, pencil, graph paper, protractor, calculator

Tasks:

9. Draw a right triangle and label the opposite, adjacent and hypotenuse on all three sides and indicate the angle θ.
10. Measure both the length (cm) and angle θ.
11. Find sin(θ), cos(θ) and tan θ.
12. Divide sin(θ) by cos(θ) and record your answers on the table.

13. Repeat the procedure with a different right triangle.
14. How are sin θ, cos θ, and tan θ related?
15. Discuss your work with other learners.

Key Takeaway:

Students should discover that $\tan \theta = \sin \theta / \cos \theta$. This relationship connects all three trigonometric ratios.

Teacher Reflection Prompts

- Did students successfully discover the two constant ratios in the anchor activity?
- Were students able to distinguish between sine and cosine?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did the real-world examples help students see the relevance of sine and cosine?
- What adjustments would improve this lesson for future classes?