I. Lesson Overview

Strand	Measurement and Geometry
Sub-Strand	Similarity and Enlargement
Specific Learning Outcome	Determining the volume scale factors of objects
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How is similarity and enlargement applied in day-to-day life?
Learning Resources	CBC Grade 10 Mathematics Textbooks

II. Learning Objectives

Category	Objective
Know	Define volume scale factor as the cube of the linear scale factor, representing the ratio by which the volume of a scaled object changes compared to the original. State the relationship: $VSF = (LSF)^3$. Distinguish between linear scale factor, area scale factor, and volume scale factor.
Do	Calculate the volume scale factor given two similar solids. Determine the volume of an image or object using the relationship $VSF = (LSF)^3$. Find the linear scale factor from a given volume scale factor by taking the cube root. Convert between volume scale factor, area scale factor, and linear scale factor.
Apply	Apply volume scale factor to solve problems involving similar cylinders, cones, containers, cubes, and real-world contexts such as architectural models, maps, and photographs.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Comparing Cuboids — What Happens to the Volume?"

Instructions (Work in pairs):

- (a) Draw a cuboid with dimensions 8 cm (length), 5 cm (width), and 4 cm (height), and label it as Cuboid A.
- (b) Draw another cuboid with dimensions 24 cm (length), 15 cm (width), and 12 cm (height), and label it as Cuboid B.
- (c) Calculate the volume of each cuboid by multiplying length × width × height.
- (d) Find the volume ratio by dividing the volume of Cuboid B by the volume of Cuboid A.
- (e) Find the ratio of the length of Cuboid B to the length of Cuboid A, then raise the result to the power of three.
- (f) Calculate the ratio by dividing the width of Cuboid B by the width of Cuboid A, then cube the resulting value.
- (g) Determine the ratio by dividing the height of Cuboid B by the height of Cuboid A, then cube the resulting value.
- (h) Compare the result obtained from step (d) with the values calculated in steps (e), (f), and (g). Note any patterns or relationships you observe among these results.
- (i) Discuss and share the results with the rest of the learners in the class.

Teacher's Role During Discovery:

- Circulate among pairs, ensuring students are calculating volumes correctly (length × width × height).
- Ask probing questions: "What is the volume of Cuboid A?" "What ratio did you get for the lengths?" "What happened when you cubed it?"
- For struggling pairs: "Remember, volume of a cuboid is length times width times height. What is 8 × 5 × 4?"
- For early finishers: "What is the ratio of the SURFACE AREAS of the two cuboids? How does that compare to the squared length ratio?"
- Guide students toward noticing that the volume ratio equals the CUBE of the linear scale factor.
- Identify 2–3 pairs with clear findings to share with the class.

Expected Student Discoveries:

- Volume of Cuboid A = $8 \times 5 \times 4 = 160$ cm^3.
- Volume of Cuboid B = $24 \times 15 \times 12 = 4{,}320$ cm^3.
- Volume ratio: $4{,}320 \div 160 = 27$.
- Length ratio: $24 \div 8 = 3$. Cubed: $3^3 = 27$.
- Width ratio: $15 \div 5 = 3$. Cubed: $3^3 = 27$.
- Height ratio: $12 \div 4 = 3$. Cubed: $3^3 = 27$.
- All four results are the same (27). The volume ratio equals the cube of the linear scale factor.
- When each dimension triples, the volume increases by a factor of 27 (= 3^3), NOT by 3.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

Definition:

A volume scale factor is the cube of the linear scale factor, representing the ratio by which the volume of a scaled object changes compared to the original object.

Key Formulas:

Formula	Description
Volume Scale Factor (VSF) = (Linear Scale Factor)^3	The volume scale factor is the cube of the linear scale factor.
VSF = Volume of Image ÷ Volume of Object	The volume scale factor can be found by dividing the volume of the image by the volume of the object.
Linear Scale Factor = $\sqrt[3]{\text{(Volume Scale Factor)}}$	To find the linear scale factor from the volume scale factor, take the cube root.
Volume of Image = VSF × Volume of Object	To find the volume of the image, multiply the volume of the object by the volume scale factor.

Complete Scale Factor Summary:

Dimension	Scale Factor	Relationship to LSF (k)
Length (1D)	Linear Scale Factor (LSF)	k
Area (2D)	Area Scale Factor (ASF)	k^2
Volume (3D)	Volume Scale Factor (VSF)	k^3

Connecting to Student Discoveries:

- Reference the cuboids: "You found that when each dimension tripled (factor of 3), the volume increased by 27. That's because $3^3 = 27$."
- Emphasise: "The volume does NOT scale by the same factor as the length. It scales by the CUBE of the linear factor."
- Connect to prior learning: "Last lesson we learned that area scales by k^2. Now we see that volume scales by k^3."
- Address common misconception: "If the linear scale factor is 2, the volume scale factor is NOT 2 and NOT 4 — it is $2^3 = 8$."
- Show the reverse: "If the volume scale factor is 64, the linear scale factor is $\sqrt[3]{64} = 4$."

Phase 3: Practice and Application (10 minutes)

Problem 1: Similar Cylinders

The corresponding heights of two similar cylinders are 4 m and 5 m.

- (a) Find the ratio of their corresponding volumes.
- (b) If the smaller cylinder has a volume of 1,536 m^3, find the volume of the larger cylinder.

Solution:

Step	Working
(a) Ratio of heights	4/5
Linear Scale Factor	4/5
Volume Scale Factor	$(4/5)^3 = 64/125$
Ratio of volumes	64 : 125
(b) Volume of larger	1536 / Volume of larger = 64/125 Volume of larger = (125 × 1536) / 64 = 3,000 m^3

Problem 2: Similar Containers — Heights and Areas from Volumes

The capacity of two similar containers are 288 cm^3 and 4,500 cm^3. Find the ratio of their:

- (a) Heights
- (b) If the area of the smaller container is 140 cm^2, find the area of the larger container.

Solution:

(a) Volume scale factor = 288/4500 = 8/125.

Linear scale factor = $\sqrt[3]{(8/125)} = 2/5$.

Therefore the ratio of the heights = 2 : 5.

(b) Area scale factor = $(2/5)^2 = 4/25$.

Area of smaller / Area of larger = 4/25.

4/25 = 140 / Area of larger container.

Area of larger container = (25 × 140) / 4 = 875 cm^2.

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. Two similar containers have heights of 6 cm and 9 cm, respectively. If the smaller container holds 400 ml, what is the capacity of the larger container?

2. Two similar cans have volumes of 192 cm^3 and 648 cm^3 respectively. If the smaller can has a height of 14 cm, what is the height of the larger can?

3. The ratio of the lengths of the corresponding sides of two similar rectangular tanks is 3 : 5. The volume of the smaller tank is 8 cm^3. Calculate the volume of the larger tank.

4. A small cube has a length of 3 cm. A larger cube is created by scaling the small cube, such that each side of the larger cube is 6 times the length of the corresponding side of the small cube.

- (a) What is the volume of the small cube?
- (b) What is the volume of the larger cube?
- (c) By what factor has the volume increased when the small cube is scaled to the larger cube?

Answer Key:

- 1. LSF = 6/9 = 2/3 (smaller to larger). VSF = $(2/3)^3 = 8/27$. But we want larger from smaller: 400 / Capacity of larger = 8/27. Capacity of larger = (27 × 400) / 8 = 1,350 ml.
- 2. VSF = 192/648 = 8/27. LSF = $\sqrt[3]{(8/27)}$ = 2/3. Height of smaller / Height of larger = 2/3. 14 / Height of larger = 2/3. Height of larger = (3 × 14) / 2 = 21 cm.

- 3. LSF = 3/5 (smaller to larger). VSF = $(3/5)^3 = 27/125$. Volume of smaller / Volume of larger = 27/125. 8 / Volume of larger = 27/125. Volume of larger = $(125 \times 8) / 27 = 1000/27 \approx 37.04$ cm^3.
- 4(a) Volume of small cube = $3^3 = 27$ cm^3.
- 4(b) Side of larger cube = $6 \times 3 = 18$ cm. Volume = $18^3 = 5{,}832$ cm^3.
- 4(c) Volume factor = 5,832 / 27 = 216. Check: LSF = 6, VSF = $6^3 = 216$. ✔

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide a summary card with all three scale factor formulas: LSF = k, ASF = k^2, VSF = k^3. Use physical cubes (e.g., sugar cubes or unit cubes) to build a 2×2×2 cube and a 4×4×4 cube so students can count and compare. Start with integer scale factors (2, 3) before fractions. Walk through Problem 1 step-by-step with the pair before independent work.
On-Level Learners	Complete all anchor activity steps and practice problems independently. Work through both forward problems (LSF → VSF → Volume) and reverse problems (Volume ratio → VSF → LSF). Use the digital textbook interactive checkpoints (2.1.33–2.1.37) for additional practice.
Advanced Learners	Extension Activity: Solve real-world problems involving architectural models, map scales, photograph enlargements, and similar triangles (shadow problems). Investigate: If the volume of a solid increases by a factor of 1000, by what factor does each linear dimension increase? ($\sqrt[3]{1000} = 10$). Challenge: A swimming pool model has volume 0.5 litres. The actual pool has LSF = 50. Find the actual volume in litres.

VI. Extension Activity

Real-World Applications of Scale Factors:

1. An architect is creating a scale model of a building. The actual height of the building is 120 meters, and the height of the model is 0.6 meters.

- (a) What is the scale factor of the model?
- (b) If the width of the actual building is 50 meters, what is the width of the model?

2. A map scale is given as 1 : 25,000, meaning 1 cm on the map represents 25,000 cm in real life.

- (a) A river on the map measures 8 cm in length. What is the actual length of the river in kilometres?
- (b) If a road on the map measures 12.5 cm, how long is the actual road in metres?

3. A photograph has a size of 5 cm by 7 cm. It needs to be enlarged so that the width becomes 20 cm. The height will also increase proportionally. What is the new height of the photograph after the enlargement?

4. A pole of height 2.4 metres casts a shadow of length 1.6 metres. A tree casts a shadow of length 12 metres.

- (a) Using the concept of similar triangles, find the height of the tree.
- (b) If the tree's shadow increases to 15 metres, what would be the new height of the tree, assuming the proportion remains the same?

Extension Answer Key:

- 1(a) Scale factor = 0.6 / 120 = 1/200 (or 1 : 200).
- 1(b) Width of model = 50 × (1/200) = 0.25 m = 25 cm.
- 2(a) Actual length = 8 × 25,000 = 200,000 cm = 2,000 m = 2 km.
- 2(b) Actual length = 12.5 × 25,000 = 312,500 cm = 3,125 m.
- 3. Scale factor = 20/5 = 4. New height = 4 × 7 = 28 cm.
- 4(a) Height/Shadow = 2.4/1.6 = 3/2. Tree height = (3/2) × 12 = 18 m.
- 4(b) Tree height = (3/2) × 15 = 22.5 m.

VII. Assessment Methods

Type	Method
Formative	Observation during pair work: Are students calculating volumes correctly? Do they see that the volume ratio equals the cubed length ratio? Questioning: "If each dimension doubles, what happens to the volume?" "Is the volume scale factor the same as the linear scale factor?" Monitoring calculations during practice problems.

Summative	Exit ticket with 4 questions covering: capacity of similar containers, height from volume ratio, volume of similar tanks, and cube scaling with volume factor. Complete answer key provided for marking.

VIII. Teacher Reflection

1. Did students discover through the anchor activity that the volume ratio equals the cube of the linear scale factor?

2. Were students able to articulate the difference between linear, area, and volume scale factors?

3. How effectively did the cuboid comparison build on prior knowledge of area scale factor?

4. Did students grasp the reverse process: finding LSF from VSF by taking the cube root?

5. Were students able to convert between VSF, ASF, and LSF within a single problem (e.g., Problem 2)?

6. What common misconceptions arose (e.g., confusing VSF with ASF or LSF)?

7. What adjustments would improve the lesson for future delivery?