Step by Step Guide: Enlargement — Negative Scale Factor

Pre-Class Preparation Checklist

- Ensure each pair has graph paper, rulers, pencils, and coloured pens (blue and red).
- Prepare a large coordinate grid on the board or projector for demonstration.
- Have the digital textbook section open: innodems.github.io/CBC-Grade-10-Maths/subsec-enlargement.html
- Prepare printed handouts with the anchor activity instructions.
- Have pre-drawn coordinate grids with triangle ABC already plotted for struggling learners.
- Write on a card: "Negative scale factor = Opposite side + Inverted".
- Prepare the comparison table (positive vs negative scale factors) on a poster or slide.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening and Recall (2 minutes)

[SAY] "In our last lesson, we explored enlargement with positive scale factors. Who can remind us what happens when we enlarge a shape with a positive scale factor?"

[WAIT] Allow students to respond. Expected: "The image gets bigger/smaller but stays on the same side."

[SAY] "Excellent! The image stays on the same side of the centre and keeps the same orientation. But what if the scale factor is NEGATIVE? What do you think would happen?"

[WAIT] Allow 30 seconds for predictions.

[SAY] "Great guesses! Let's investigate and find out. Today we will discover what a negative scale factor does to a shape."

Anchor Activity Launch (3 minutes)

[SAY] "Work in pairs. Each pair needs graph paper, a ruler, and two coloured pens — blue and red."

[DO] Distribute materials and display the instructions.

[SAY] "First, draw your axes and plot triangle ABC with vertices A at (2, 1), B at (4, 1), and C at (3, 3). Use blue to draw the triangle."

[SAY] "Now here is the challenge: For each vertex, draw a line from the point THROUGH the origin O, and extend it to the OTHER SIDE of O."

[SAY] "On the opposite side of O, mark a new point at the SAME distance from O as the original point. Use red for these new points."

[SAY] "Label the new points A-prime, B-prime, C-prime and connect them in red to form a new triangle."

[SAY] "Then answer these questions: What are the coordinates of A-prime, B-prime, C-prime? What pattern do you see? Is the image the same way up or inverted?"

Student Work Time (8 minutes)

[DO] Circulate among pairs. Check that students are drawing lines THROUGH O, not stopping at O.

[ASK] To pairs plotting image points: "Where is A-prime? Is it on the same side of O as A, or the opposite side?"

[DO] For struggling pairs: "Let me help. A is at (2, 1). Draw a line from A through O. Now continue the line the same distance past O. Where do you end up? At (−2, −1)."

[ASK] To pairs who found coordinates: "What happened to each coordinate? Can you see a pattern?"

[ASK] "Look at the two triangles. Is the image the same way up as the original?"

[ASK] To advanced pairs: "What if you marked points at TWICE the distance on the opposite side? What scale factor would that be?"

[DO] Note which pairs have clear explanations for the sharing phase.

Class Sharing (2 minutes)

[SAY] "Let's hear from a few pairs. What coordinates did you get for A-prime, B-prime, and C-prime?"

[WAIT] Call on 2 pairs to share.

[SAY] "A-prime is at (−2, −1), B-prime is at (−4, −1), and C-prime is at (−3, −3). Each coordinate became its negative!"

[ASK] "And what about the orientation? Is the image the same way up?"

[SAY] "No! The image is INVERTED — it's upside down and on the opposite side of the centre. This is exactly what a NEGATIVE scale factor does."

Phase 2: Structured Instruction (10 minutes)

Formalising the Negative Scale Factor (4 minutes)

[SAY] "Let's formalise what you discovered. When an enlargement has a NEGATIVE scale factor, two things happen."

[WRITE] On the board: "Negative Scale Factor Rule:"

[WRITE] "1. The image is formed on the OPPOSITE side of the centre."

[WRITE] "2. The image is INVERTED (upside down)."

[SAY] "In your activity, the scale factor was negative one. Each coordinate was multiplied by negative one: A(2,1) became A-prime(−2, −1)."

[WRITE] "For centre at origin: Image of (x, y) with scale factor k = (kx, ky)"

[SAY] "This formula works for both positive AND negative scale factors. When k is negative, the signs of both coordinates flip — that's what puts the image on the opposite side."

Comparison Table (3 minutes)

[SAY] "Let me show you a comparison that makes the difference crystal clear."

[WRITE] Draw the comparison table on the board:

[SAY] "With a POSITIVE scale factor, the image stays on the same side and keeps the same orientation. With a NEGATIVE scale factor, the image moves to the opposite side and flips upside down."

[SAY] "The ABSOLUTE VALUE of the scale factor still controls the size. If k = −2, the image is twice as big AND inverted. If k = negative one-half, the image is half the size AND inverted."

[ASK] "So does 'negative' mean the shape gets smaller?"

[WAIT] Students respond.

[SAY] "No! Negative controls the DIRECTION — opposite side and inverted. The absolute value controls the SIZE."

Non-Origin Centre (3 minutes)

[SAY] "What if the centre of enlargement is NOT the origin? We use a slightly different formula."

[WRITE] "Image = Centre + k × (Point − Centre)"

[SAY] "For example, if the centre is at (1, 1) and we enlarge point X(4, 0) by scale factor −2:"

[WRITE] "X' = (1 + (−2)(4−1), 1 + (−2)(0−1))"

[WRITE] "X' = (1 + (−2)(3), 1 + (−2)(−1))"

[WRITE] "X' = (1 − 6, 1 + 2) = (−5, 3)"

[SAY] "Notice: we subtract the centre first, multiply by k, then add the centre back. This works for any centre and any scale factor."

Phase 3: Practice and Application (10 minutes)

Worked Example 1: Construction Method (3 minutes)

[SAY] "Let's work through Problem 1 together. We need to enlarge triangle ABC by scale factor −1 about the origin."

[DO] Draw the triangle on the board grid.

[SAY] "Step 1: Draw a line from A through O. Step 2: Measure OA = 5 units. Step 3: Since k = −1, OA-prime = |−1| × 5 = 5 units, but on the OPPOSITE side of O."

[DO] Mark A' on the opposite side at the same distance.

[SAY] "Repeat for B and C. Then connect A-prime, B-prime, C-prime."

[SAY] "The image is the same size but inverted and on the opposite side. Scale factor −1 is like rotating 180 degrees about the centre."

Worked Example 2: Circle Enlargement (2 minutes)

[SAY] "Problem 2: A circle has radius 22.4 cm and is enlarged by scale factor 0.25. Find the circumference of the image."

[WRITE] "New radius = 0.25 × 22.4 = 5.6 cm"

[WRITE] "Circumference = 2πr = 2 × 22/7 × 5.6 = 35.2 cm"

[SAY] "For circles, the radius scales by the scale factor, and so does the circumference. The area would scale by k-squared."

Student Practice (5 minutes)

[SAY] "Now try Problem 3 on your own. Triangle ABC has vertices A(−2, 9), B(8, 7), C(5, 4). Enlarge with origin as centre and scale factor one-half."

[DO] Give students 3 minutes to calculate. Circulate and assist.

[SAY] "Remember: multiply each coordinate by the scale factor."

[DO] After 3 minutes, reveal the answers:

[WRITE] "A'(−1, 9/2), B'(4, 7/2), C'(5/2, 2)"

[ASK] "Is this image larger or smaller than the original? On which side of the centre?"

[SAY] "Smaller, because the scale factor is one-half. And it's on the SAME side because the scale factor is positive."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these six questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: Triangle XYZ has vertices X(4,0), Y(6,3), Z(5,4). Centre of enlargement is (1,1). Find the image with scale factor (a) −2 and (b) one-half."

[SAY] "Question 2: Triangle ABC with A(2,6), B(4,6), C(4,2). Centre is (0,2). Find the image with scale factor −1."

[SAY] "Question 3: Triangle PQR with P(1,4), Q(3,4), R(3,1). Origin as centre. Find image with scale factor (a) negative one-quarter, (b) −3, (c) 2."

[SAY] "Question 4: A rectangle is 5 cm by 9 cm. Find the image dimensions after enlargement with scale factor −2."

[SAY] "Question 5: A photograph width increases from 10 cm to 25 cm. Original height is 15 cm. Find the new height."

[SAY] "Question 6: A map scale is 1:50,000. Distance on map is 8 cm. Find the actual distance."

[DO] Collect exit tickets as students finish.

Answer Key:

- 1(a) k=−2, centre (1,1): X'(−5, 3), Y'(−9, −3), Z'(−7, −5)
- 1(b) k=½, centre (1,1): X'(2.5, 0.5), Y'(3.5, 2), Z'(3, 2.5)
- 2. k=−1, centre (0,2): A'(−2, −2), B'(−4, −2), C'(−4, 2)
- 3(a) k=−¼: P'(−¼, −1), Q'(−¾, −1), R'(−¾, −¼)
- 3(b) k=−3: P'(−3, −12), Q'(−9, −12), R'(−9, −3)
- 3(c) k=2: P'(2, 8), Q'(6, 8), R'(6, 2)
- 4. Dimensions: 10 cm by 18 cm (image is inverted).
- 5. Scale factor = 25/10 = 2.5. New height = 37.5 cm.
- 6. Actual distance = 8 × 50,000 = 400,000 cm = 4 km.

Differentiation Notes

Struggling Learners:

Provide pre-drawn grids with triangle ABC plotted. Use colour coding: blue for object lines, red for image lines. Start with scale factor −1 only. Provide step-by-step checklist: (1) Draw line through centre, (2) Measure distance, (3) Mark same distance on opposite side, (4) Repeat, (5) Connect.

On-Level Learners:

Complete all problems independently using both construction and coordinate methods. Use digital textbook checkpoints for additional practice. Encourage peer explanation of the positive vs negative difference.

Advanced Learners:

Investigate successive enlargements: prove that two enlargements with scale factors k_1 and k_2 about the same centre equal one enlargement with scale factor $k_1 \times k_2$. Explore area scaling with negative scale factors (area = k^2 × original area). Investigate what happens when the centre of enlargement is INSIDE the shape.

Post-Lesson Reflection

1. Did students understand that a negative scale factor places the image on the opposite side of the centre?

2. Were students able to distinguish between the effect of the sign (direction) and the absolute value (size)?

3. How effectively did the anchor activity build on prior knowledge of positive scale factor enlargement?

4. Did students grasp the non-origin centre formula: Image = Centre + k(Point − Centre)?

5. Were the colour-coded construction steps helpful for visual learners?

6. How well did the comparison table clarify the positive vs negative distinction?

7. What adjustments would improve the lesson for future delivery?