Step by Step Guide: Properties of Rotation

Pre-Class Preparation Checklist

- Prepare graph paper for each pair of students.
- Ensure rulers, protractors, and pencils are available for every pair.
- Print handouts with the pre-drawn triangles ABC and A′B′C′ (from the textbook figure) for the anchor activity.
- Prepare a recording table handout for students to fill in distances and angles.
- Write the rotation rules table on chart paper or prepare to build it on the board.
- Have compasses available for the perpendicular bisector construction.
- Have the digital textbook open: Section 2.3.1 Properties of Rotation.
- Prepare exit ticket handouts with 5 questions.
- Optional: Bring a small clock, a fidget spinner, or a picture of a Ferris wheel for the real-life connection.

Phase 1: Problem-Solving and Discovery (15 minutes)

Opening Hook (2 minutes)

[DO] Hold up a clock (or show a picture of one).

[SAY] "What is the minute hand doing right now?"

[WAIT] Expected: "Moving." "Turning." "Rotating."

[SAY] "Exactly — it's ROTATING. It turns around a fixed point — the centre of the clock. Today, we're going to discover the mathematical PROPERTIES of rotation."

[ASK] "Can you think of other things that rotate in real life?"

[WAIT] Expected: "Wheels." "Ferris wheel." "Windmill." "Fan." "Earth."

[SAY] "Great examples! All of these involve rotation around a fixed point. Let's investigate what happens mathematically when we rotate a shape."

Anchor Activity Launch (3 minutes)

[DO] Distribute graph paper, rulers, protractors, and the handout with pre-drawn triangles.

[SAY] "On your handout, you can see triangle ABC and its image triangle A′B′C′. The image was created by ROTATING the original triangle."

[SAY] "Your job is to discover the PROPERTIES of this rotation. Here's what I want you to do."

[SAY] "Step 1: Pick a point on the graph to act as the centre of rotation. Mark it O."

[SAY] "Step 2: Draw lines from each vertex to O, and from each image vertex to O."

[SAY] "Step 3: Measure the distances OA and OA′, OB and OB′, OC and OC′. Record them in your table."

[SAY] "Step 4: Using your protractor, measure the angles ∠AOA′, ∠BOB′, and ∠COC′. Record them."

[SAY] "Step 5: Look at your results. What patterns do you notice?"

[SAY] "Work in pairs. You have 8 minutes."

Student Work Time (8 minutes)

[DO] Circulate among pairs.

[ASK] To a pair measuring distances: "What did you get for OA and OA′?"

[WAIT] Expected: "They're the same!" or specific measurements.

[ASK] "What about OB and OB′?"

[WAIT] Expected: "Also the same!"

[SAY] "Interesting! Write that observation down."

[ASK] To another pair measuring angles: "What did you get for ∠AOA′?"

[WAIT] Expected: "90 degrees."

[ASK] "And ∠BOB′?"

[WAIT] Expected: "Also 90 degrees!"

[SAY] "So the angle is the same for all three pairs of points. That's a very important discovery!"

[DO] For struggling pairs: "Let me help you measure. Place the ruler from A to O. Read the distance. Now from A′ to O. Compare them."

[DO] For early finishers: "What if you chose a DIFFERENT point as the centre? Would the distances still be equal? Would the angles still be equal?"

Class Sharing (2 minutes)

[SAY] "Let's share your discoveries. What did you notice about the distances?"

[WAIT] Expected: "OA equals OA'." "All the distances are the same."

[SAY] "And what about the angles?"

[WAIT] Expected: "They're all 90 degrees." "The angles are all equal."

[SAY] "Excellent! You've just discovered the TWO key properties of rotation. Let's formalise them."

Phase 2: Structured Instruction (10 minutes)

Formalising the Properties (4 minutes)

[SAY] "You discovered two important properties. Let me write them formally."

[WRITE] "Property 1: The distance from any point to the centre of rotation equals the distance from its image to the centre."

[WRITE] "AO = A'O, BO = B'O, CO = C'O"

[SAY] "Every point stays the same distance from the centre. The shape doesn't move closer to or farther from the centre — it just TURNS."

[WRITE] "Property 2: The angle of rotation is the same for all points in the shape."

[WRITE] "$\angle AOA' = \angle BOB' = \angle COC' = 90°$"

[SAY] "Every point turns through the SAME angle. In this case, 90 degrees. Point O is the centre of rotation and 90° is the angle of rotation."

[SAY] "These two properties mean that rotation preserves the shape and size — the image is CONGRUENT to the original."

Sign Convention and Notation (3 minutes)

[SAY] "Now, an important convention about direction."

[WRITE] "Anticlockwise = POSITIVE. Example: +45° means 45° anticlockwise."

[WRITE] "Clockwise = NEGATIVE. Example: −45° means 45° clockwise."

[SAY] "Think of it this way: anticlockwise is the 'positive' direction in mathematics, just like moving up or to the right on a number line."

[ASK] "If I say a rotation of +90°, which direction is that?"

[WAIT] Expected: "Anticlockwise."

[ASK] "And −90°?"

[WAIT] Expected: "Clockwise."

[SAY] "One more important point: for a rotation to be COMPLETELY defined, you must state TWO things: the CENTRE of rotation and the ANGLE of rotation (including direction)."

Rotation Rules on the Coordinate Plane (3 minutes)

[SAY] "When the centre of rotation is the origin, we have simple rules."

[WRITE] Build the rules table on the board:

[WRITE] "90° anticlockwise (+90°): (x, y) → (−y, x)"

[WRITE] "90° clockwise (−90°): (x, y) → (y, −x)"

[WRITE] "180°: (x, y) → (−x, −y)"

[SAY] "Let's test: if A is at (3, 2) and we rotate 90° clockwise, the rule is (x, y) → (y, −x)."

[WRITE] "A(3, 2) → A′(2, −3)"

[ASK] "Can someone verify: is OA = OA′?"

[WAIT] Expected: "OA = √(9+4) = √13, OA′ = √(4+9) = √13. Yes!"

[SAY] "The distance property holds — as it always does with rotation."

Phase 3: Practice and Application (10 minutes)

Worked Example: Rotating Triangle PQR (3 minutes)

[SAY] "Let's work through the textbook example. Triangle PQR has vertices P(−8, −6), Q(−2, −6), R(−5, −3). We rotate 90° clockwise about the origin."

[SAY] "Which rule do we use?"

[WAIT] Expected: "(x, y) → (y, −x)."

[SAY] "Correct!"

[WRITE] "P(−8, −6) → P′(−6, 8)"

[WRITE] "Q(−2, −6) → Q′(−6, 2)"

[WRITE] "R(−5, −3) → R′(−3, 5)"

[SAY] "Let's verify the distance property for P."

[WRITE] "OP = √(64 + 36) = √100 = 10"

[WRITE] "OP′ = √(36 + 64) = √100 = 10 ✔"

[SAY] "OP equals OP′. The property holds!"

Student Practice (4 minutes)

[SAY] "Now try these two problems on your own."

[WRITE] "Problem 1: Square A(1,1), B(4,1), C(4,4), D(1,4). Rotate 90° anticlockwise about the origin."

[WRITE] "Problem 2: Triangle L(2,3), M(5,1), N(6,5). Rotate 180° about the origin."

[SAY] "Work individually first, then check with your partner. 3 minutes."

[WAIT] 3 minutes.

[SAY] "Let's check Problem 1. Which rule?"

[WAIT] Expected: "(x, y) → (−y, x) for 90° anticlockwise."

[SAY] "So A(1,1) becomes?"

[WAIT] Expected: "A′(−1, 1)."

[SAY] "And B(4,1)?"

[WAIT] Expected: "B′(−1, 4)."

[SAY] "Problem 2: 180° rotation rule?"

[WAIT] Expected: "(x, y) → (−x, −y)."

[SAY] "So L(2,3) becomes L′(−2, −3). Correct!"

Finding Centre and Angle of Rotation (3 minutes)

[SAY] "Sometimes you're given the original and the image, and you need to FIND the centre and angle. Here's the method."

[WRITE] "Step 1: Join a point to its image (e.g., Z to Z′)."

[WRITE] "Step 2: Construct the perpendicular bisector of ZZ′."

[WRITE] "Step 3: Join another point to its image (e.g., Y to Y′)."

[WRITE] "Step 4: Construct the perpendicular bisector of YY′."

[WRITE] "Step 5: The intersection = centre of rotation O."

[WRITE] "Step 6: Join Z and Z′ to O. Measure $\angle ZOZ'$ = angle of rotation."

[SAY] "From the textbook example, the centre was (−1, 1) and the angle was −160° — that's 160° clockwise."

Phase 4: Assessment — Exit Ticket (5 minutes)

[SAY] "For our exit ticket, answer these five questions on a separate piece of paper. You have 5 minutes."

[SAY] "Question 1: State the two key properties of rotation."

[SAY] "Question 2: Rotate triangle A(3,2), B(6,2), C(5,5) through 90° clockwise about the origin."

[SAY] "Question 3: Describe how you would find the centre and angle of rotation by construction."

[SAY] "Question 4: A rotation of +135° means the shape has been rotated how many degrees in which direction?"

[SAY] "Question 5: Give two real-life examples of rotation and identify the centre of rotation."

[DO] Collect exit tickets as students finish.

Answer Key:

- 1. (i) Distance from point to centre = distance from image to centre. (ii) Angle of rotation is the same for all points.
- 2. A′(2, −3), B′(2, −6), C′(5, −5).

- 3. Join corresponding points, construct perpendicular bisectors, intersection = centre. Measure angle at centre.
- 4. 135° in the anticlockwise direction.
- 5. Clock hands (centre = pivot), Ferris wheel (centre = axle), windmill (centre = hub), etc.

Differentiation Notes

Struggling Learners:

Provide pre-drawn figures with centre marked. Use the recording table for structured measurements. Focus on 90° clockwise rule only. Allow tracing paper for physical rotation. Pair with stronger students. Provide reference card with rotation rules.

On-Level Learners:

Complete all practice problems. Verify distance property using distance formula. Use perpendicular bisector method. Identify real-life examples with centre and angle.

Advanced Learners:

Investigate 360° and 720° rotations. Explore composition of rotations. Find centre for non-origin rotations. Prove congruence preservation. Investigate rotation-reflection relationship.

Post-Lesson Reflection

1. Did the measuring activity help students discover the two key properties?

2. Were students able to measure distances and angles accurately?

3. Did students understand positive (anticlockwise) vs. negative (clockwise)?

4. Were students able to apply rotation rules on the coordinate plane?

5. Did students grasp the perpendicular bisector method?

6. Were students able to connect rotation to real-life examples?

7. What common errors arose (e.g., sign confusion, wrong rule)?

8. What adjustments would improve the lesson for future delivery?