I. Lesson Overview

Strand	Measurement and Geometry
Sub-Strand	Reflection and Congruence
Specific Learning Outcome	Carry out congruence tests for triangles
Grade Level	Grade 10
Duration	40 minutes
Key Inquiry Question	How is similarity and enlargement applied in day-to-day life? What conditions must be met for two triangles to be congruent?
Learning Resources	CBC Grade 10 Mathematics Textbooks, construction paper, pencil, ruler, protractor, scissors

II. Learning Objectives

Category	Objective
Know	Define congruence in triangles. State the five congruence criteria: SSS, SAS, ASA, RHS, and AAS. Recall that congruent triangles have all corresponding sides and angles equal. Distinguish between direct congruence and opposite congruence. Understand that the order of vertices matters when writing congruence statements.
Do	Identify pairs of congruent triangles from a set of given triangles. Determine which congruence criterion applies to a given pair of triangles. Write formal congruence statements using the ≅ symbol with correct vertex correspondence. Measure sides and angles to verify congruence. Construct congruent triangles using ruler, protractor, and compass.
Apply	Prove that two triangles are congruent by identifying matching sides and angles. Apply congruence tests to reflected triangles on the Cartesian plane. Use congruence criteria to solve problems involving equilateral triangles and midpoints. Recognise congruence in real-world structures such as bridges, roofs, and tiled patterns.

III. Materials & Resources

- CBC Grade 10 Mathematics Textbooks
- Construction paper
- Pencils, rulers, and protractors
- Scissors (for cutting out triangles)
- Printed handouts with sets of triangles for the anchor activity
- Graph paper (for the coordinate geometry assessment questions)
- Coloured pencils or markers
- Compass (for constructing equilateral triangles)

IV. Lesson Procedure

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: "Conditions for Congruence in Triangles"

Materials: Construction paper, pencil, ruler, protractor

Instructions (Work in groups):

- Trace the given triangles on construction paper.
- Cut out the triangles carefully.
- Identify pairs of congruent triangles by placing one on top of the other.
- From the pairs of congruent triangles you have identified, determine which pairs fit the following criteria:

Criterion Number	Description
Criterion 1	The three sides of one triangle are equal to the three sides of the corresponding triangle.
Criterion 2	Two sides and an included angle of one triangle are equal to the two corresponding sides and the included angle of the other triangle.
Criterion 3	One side and two included angles of one triangle are equal to the corresponding side and the two included angles of the other triangle.
Criterion 4	One side and the hypotenuse of a right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle.

Teacher's Role During Discovery:

- Circulate among groups, ensuring students trace and cut triangles accurately.
- Ask probing questions: "How did you decide those two triangles are congruent?" "What measurements did you compare?"
- For struggling groups: "Start by measuring all three sides of each triangle. Can you find two triangles where all three sides match?"
- For early finishers: "Can you find a pair that does NOT fit any of the four criteria? Why not?"
- Guide students to notice: some pairs match by sides only (SSS), some by two sides and the angle between them (SAS), some by a side and two angles (ASA), and some by the hypotenuse and a side in right-angled triangles (RHS).
- Identify 2–3 groups with clear findings to share with the class.

Expected Student Discoveries:

Observation	Mathematical Significance
When all three sides match, the triangles are identical.	This is the SSS (Side-Side-Side) criterion.
When two sides and the angle between them match, the triangles are identical.	This is the SAS (Side-Angle-Side) criterion. The angle must be the INCLUDED angle.
When one side and two angles match, the triangles are identical.	This is the ASA (Angle-Side-Angle) criterion.
In right-angled triangles, when the hypotenuse and one other side match, they are identical.	This is the RHS (Right angle-Hypotenuse-Side) criterion.

Phase 2: Structured Instruction (10 minutes)

Key Takeaways:

Congruence in triangles depends on the measure of the sides and angles. Two triangles are said to be congruent if all pairs of corresponding sides and corresponding angles are equal.

Congruence Criteria Summary Table:

Criterion	Abbreviation	What Must Be Equal
Side-Side-Side	SSS	All three sides of one triangle are equal to the three corresponding sides of the other triangle.

Side-Angle-Side	SAS	Two sides and the INCLUDED angle (the angle between those two sides) of one triangle are equal to the corresponding two sides and included angle of the other triangle.
Angle-Side-Angle	ASA	Two angles and the INCLUDED side (the side between those two angles) of one triangle are equal to the corresponding two angles and included side of the other triangle.
Right angle-Hypotenuse-Side	RHS	Both triangles have a right angle, and the hypotenuse and one other side of one triangle are equal to the hypotenuse and corresponding side of the other triangle.
Angle-Angle-Side	AAS	Two angles and a NON-INCLUDED side of one triangle are equal to the corresponding two angles and non-included side of the other triangle.

Important Notes:

- The congruence symbol is ≅. For example, △ABC ≅ △PQR means triangle ABC is congruent to triangle PQR.
- The ORDER of vertices matters: △ABC ≅ △PQR means A corresponds to P, B corresponds to Q, and C corresponds to R.
- Direct congruence: one triangle fits directly on top of the other without flipping.
- Opposite congruence: one triangle must be flipped (laterally inverted) to fit on the other. Reflected triangles have opposite congruence.
- AAA (Angle-Angle-Angle) is NOT a congruence criterion — it only proves similarity, not congruence.
- SSA (Side-Side-Angle) where the angle is NOT included is NOT a valid congruence criterion (the ambiguous case).

Connecting to Student Discoveries:

- Reference the anchor activity: "In your groups, you matched triangles by measuring sides and angles. Now we have formal names for each matching criterion."
- Address misconception: "Remember, for SAS the angle MUST be between the two sides. If the angle is not between the two sides, the criterion does not guarantee congruence."

- Emphasise: "You discovered that you don't need to check ALL sides and ALL angles. Just the right combination is enough to prove congruence."

Phase 3: Practice and Application (10 minutes)

Problem 1 (Worked Example): Checking Congruence Using SAS

Check if the triangles below are congruent and state the test of congruence criterion.

Given: Triangle ABC and Triangle PQR, where AB = PR = 3 cm, BC = PQ = 8 cm, and ∠B = ∠P = 60°.

Solution:

Step 1: Identify corresponding sides and angles from the figure.

Step 2: AB = PR = 3 cm (first pair of equal sides).

Step 3: BC = PQ = 8 cm (second pair of equal sides).

Step 4: ∠B = ∠P = 60° (the angle between the two equal sides — the included angle).

Step 5: Two sides and the included angle are equal.

Therefore, △ABC ≅ △PQR by the SAS criterion.

Problem 2: Proving Congruence Using SSS

Triangle DEF has sides DE = 5 cm, EF = 7 cm, and DF = 9 cm. Triangle GHI has sides GH = 5 cm, HI = 7 cm, and GI = 9 cm. Are the triangles congruent? State the criterion.

Solution:

DE = GH = 5 cm, EF = HI = 7 cm, DF = GI = 9 cm.

All three pairs of corresponding sides are equal.

Therefore, △DEF ≅ △GHI by the SSS criterion.

Problem 3: Proving Congruence Using ASA

In triangles LMN and XYZ: ∠L = ∠X = 50°, LM = XY = 6 cm, and ∠M = ∠Y = 70°. Are the triangles congruent? State the criterion.

Solution:

∠L = ∠X = 50° (first pair of equal angles).

LM = XY = 6 cm (the side between the two angles — the included side).

∠M = ∠Y = 70° (second pair of equal angles).

Two angles and the included side are equal.

Therefore, △LMN ≅ △XYZ by the ASA criterion.

Problem 4: Proving Congruence Using RHS

Triangle PQR is right-angled at Q with hypotenuse PR = 10 cm and QR = 6 cm. Triangle STU is right-angled at T with hypotenuse SU = 10 cm and TU = 6 cm. Are the triangles congruent?

Solution:

Both triangles are right-angled (∠Q = ∠T = 90°).

Hypotenuse: PR = SU = 10 cm.

One other side: QR = TU = 6 cm.

Therefore, △PQR ≅ △STU by the RHS criterion.

Phase 4: Assessment — Exit Ticket (5 minutes)

Assessment Questions:

1. A(0, 4), B(−3, 0) and C(0, 2) are the coordinates of △ABC. Reflect the triangle over the mirror line x = 0. Prove that the triangle and its image are congruent and state the test of congruence criterion.

2. Construct an equilateral triangle UVW with sides 6 cm. X is the midpoint of UW and VX is perpendicular to UW. Show that △UVX ≅ △VWX. State the test of congruence criterion.

Answer Key:

Question 1 Solution:

Reflecting over x = 0 (the y-axis): (x, y) ⟶ (−x, y).

A(0, 4) ⟶ A′(0, 4) [point on the y-axis maps to itself]

B(−3, 0) ⟶ B′(3, 0)

C(0, 2) ⟶ C′(0, 2) [point on the y-axis maps to itself]

Calculate side lengths using the distance formula:

$AB = \sqrt{(-3-0)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$A'B' = \sqrt{(3-0)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(0-(-3))^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13}$

$B'C' = \sqrt{(0-3)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13}$

$AC = \sqrt{(0-0)^2 + (2-4)^2} = \sqrt{4} = 2$

$A'C' = \sqrt{(0-0)^2 + (2-4)^2} = \sqrt{4} = 2$

AB = A′B′ = 5, BC = B′C′ = $\sqrt{13}$, AC = A′C′ = 2.

△ABC ≅ △A′B′C′ by the SSS criterion.

Question 2 Solution:

In equilateral triangle UVW with sides 6 cm, X is the midpoint of UW.

Therefore UX = XW = 3 cm.

VX is perpendicular to UW, so ∠VXU = ∠VXW = 90°.

VX is common to both triangles.

In △UVX and △VWX:

• ∠VXU = ∠VXW = 90° (both right angles)

• UV = VW = 6 cm (sides of equilateral triangle — hypotenuse)

• UX = XW = 3 cm (X is midpoint of UW)

△UVX ≅ △VWX by the RHS criterion (Right angle, Hypotenuse UV = VW, Side UX = XW).

V. Differentiation Strategies

Learner Level	Strategy
Struggling Learners	Provide pre-cut triangles so students can focus on comparing rather than cutting. Use colour-coded sides and angles: mark equal sides in the same colour. Start with SSS only (the most intuitive criterion) before introducing others. Provide a reference card with all five criteria and simple diagrams. Allow students to physically overlay triangles rather than working purely from measurements. Pair with a stronger student during the practice phase.
On-Level Learners	Complete all four practice problems identifying the correct criterion. Write formal congruence statements with correct vertex correspondence. Attempt the coordinate geometry problem (Question 1) using the distance formula. Verify their criterion choice by checking that the conditions are fully met.
Advanced Learners	Investigate why AAA is NOT a congruence criterion by constructing two triangles with the same angles but different sizes. Explore the ambiguous case: why SSA does not guarantee congruence (construct two different triangles with the same SSA). Prove that in an isosceles triangle, the

	altitude from the vertex angle creates two congruent triangles. Create their own congruence proof involving a quadrilateral divided into triangles by a diagonal.

VI. Extension Activity

Activity: Congruence in the Real World and Beyond

1. Show that △ABC ≅ △ADB if AD = AE = BE = BC. (Hint: Identify the common side and use the given equal lengths to establish the criterion.)

2. A rhombus PQRS has diagonals PR and QS intersecting at point O. Prove that △POQ ≅ △ROS. State the congruence criterion used.

3. In a rectangle ABCD, the diagonals AC and BD intersect at point M. Prove that △AMB ≅ △CMD. What criterion applies?

4. Real-world investigation: Identify three structures or objects in your school or community that use congruent triangles in their design (e.g., roof trusses, bridge supports, tiled floors). For each, sketch the congruent triangles and state which congruence criterion you would use to prove they are congruent.

5. Challenge: Two triangles have five pairs of equal elements (e.g., three pairs of sides and two pairs of angles). Is it possible for them NOT to be congruent? Explain your reasoning.

Extension Answer Key:

- 1. In △ABC and △ADB: AD = BC (given), AE = BE (given), and AB is common. Therefore △ABC ≅ △ADB by SSS.
- 2. In rhombus PQRS: PO = OR (diagonals of a rhombus bisect each other), QO = OS (diagonals bisect each other), ∠POQ = ∠ROS (vertically opposite angles). Therefore △POQ ≅ △ROS by SAS.

- 3. In rectangle ABCD: AM = CM (diagonals of a rectangle bisect each other), BM = DM (diagonals bisect each other), ∠AMB = ∠CMD (vertically opposite angles). Therefore △AMB ≅ △CMD by SAS.
- 4. Answers will vary. Examples: roof trusses (SSS — all beams cut to same length), bridge supports (SAS — two beams and fixed angle), floor tiles (SSS or ASA depending on tile pattern).
- 5. No, it is not possible. If five of the six elements are equal, the sixth must also be equal (by the angle sum property or the relationship between sides and angles). The triangles must be congruent.

VII. Assessment Methods

Type	Method
Formative	Observation during group work: Can students accurately trace, cut, and overlay triangles? Do they correctly identify matching sides and angles? Questioning: "Which criterion are you using?" "Why is the angle important in SAS?" "What makes RHS different from SSS?" Monitoring: Check that students write congruence statements with correct vertex correspondence.
Summative	Exit ticket with 2 questions: (1) Coordinate geometry reflection proof using SSS with distance formula calculations, and (2) Equilateral triangle construction proof using RHS. Both require formal proof structure and correct identification of the congruence criterion.

VIII. Teacher Reflection

1. Did the hands-on tracing and cutting activity effectively help students discover the congruence criteria?

2. Were students able to distinguish between the five criteria (SSS, SAS, ASA, RHS, AAS)?

3. Did students understand why the angle must be INCLUDED for SAS and the side must be INCLUDED for ASA?

4. Were students able to write congruence statements with correct vertex correspondence?

5. Did students grasp the difference between direct congruence and opposite congruence?

6. Were students able to apply congruence criteria to the coordinate geometry problem?

7. What common errors arose (e.g., confusing SAS with SSA, or forgetting vertex order)?

8. What adjustments would improve the lesson for future delivery?