Grade 10 Mathematics Lesson Plan

Using Two Sides and an Angle

Strand:	Measurement and Geometry
Sub-Strand:	Area of Polygons: Area of Triangles
Specific Learning Outcome:	Derive the formula for the area of a triangle given two sides and an included angle. Work out the area of a triangle given two sides and an included angle.
Duration:	40 minutes
Key Inquiry Question:	How do we work out the area of polygons?
Learning Resources:	CBC Grade 10 textbooks, measuring tapes, protractors, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Measuring shadow length and angle to derive area formula
Structured Instruction	10 minutes	Deriving the formula: Area = (1/2) ab sin C
Practice and Application	10 minutes	Worked examples using the formula
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Measuring Height Using Shadow and Angle of Elevation

Students work in groups to measure the shadow length of a tall object (tree, flagpole, building), measure the angle of elevation, and use trigonometry to find the height and area of the triangle formed.

Materials Required:

- A tall object (flagpole, tree or building)
- Measuring tape or ruler
- Protractor or phone app

- Calculator
- Paper and pencil
- A sunny day (for shadow measurement)

Instructions for Students:

1. Identify the Tall Object: Choose a tree, flagpole, or lamp post as the vertical height (like A in the diagram). The ground acts as the base (BC).
2. Find the Shadow Length: Measure the length of the shadow cast by the object on the ground (BC). This represents the long horizontal base in the diagram.
3. Measure the Angle of Elevation: Stand some distance away and use a protractor or a phone app to measure the angle of elevation from your eyes to the top of the object. This represents the 30 degree angle at B in the diagram. If no protractor is available, use similar triangles by measuring the shadow of a known object (like a stick) and comparing proportions.
4. Discuss your findings with your group members.

Recording Table for Student Observations:

Object Measured	Shadow Length (m)	Angle of Elevation (degrees)	Calculated Height (m)
____	___	___	___

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to measure the shadow and angle correctly.
- Ask probing questions: "What triangle are you forming?" "What is the base?" "What is the height?"
- For struggling groups: "Let us draw the triangle together. The shadow is the base, the object is the height, and the angle is here."
- For early finishers: "Can you use trigonometry to find the height without measuring it directly?"
- Guide students to articulate: "We can use sine to find the height: height = hypotenuse times sin(angle)."
- Identify 2-3 groups with clear calculations to share with the class.

Discovery Table: Linking Observations to Mathematical Significance

Student Observation	Mathematical Significance

The shadow, object, and angle form a right triangle	We can use trigonometric ratios to find the height
The height can be found using sine: h = a sin(angle)	Sine relates the opposite side (height) to the hypotenuse
The area of the triangle is (1/2) times base times height	Substituting h = b sin C gives Area = (1/2) ab sin C
We can find the area without knowing the height directly	The formula Area = (1/2) ab sin C is useful when we know two sides and the included angle
Different angles give different areas	The sine function determines how much of the two sides contributes to the area

Phase 2: Structured Instruction (10 minutes)

Deriving the Formula: Area = (1/2) ab sin C

After students have completed the anchor activity and shared their findings, the teacher formalizes the derivation of the area formula.

Step 1: Recall the Basic Formula

Area = (1/2) times Base times Height

Step 2: Consider a Triangle with an Angle

- Let us take a triangle ABC with sides a, b and included angle C.
- Side a and Side b form the triangle.
- The height h is perpendicular from the top vertex to the base.

Step 3: Express Height in Terms of Sin

Using trigonometry, we know that in a right-angled triangle:

sin(theta) = Opposite / Hypotenuse

In our case, the height h is the opposite side of angle C, and side b is the hypotenuse.

So, we can write it as: h = b sin C

Step 4: Substitute into the Area Formula

Now, take the basic Formula for finding area of a triangle:

Substituting Base = a and Height = b sin C:

Area = (1/2) times a times (b sin C)

Area = (1/2) ab sin C

NOTE:

Area = (1/2) ab sin C

This formula is useful when we know two sides of a triangle and the angle between them instead of the height.

Scaffolding Strategies to Address Misconceptions:

- Misconception: "I can use any two sides and any angle." Clarification: "No, the angle must be the included angle between the two sides."
- Misconception: "The formula only works for right triangles." Clarification: "No, it works for all triangles as long as you know two sides and the included angle."
- Misconception: "I need to know the height to find the area." Clarification: "No, the formula (1/2) ab sin C allows you to find the area without knowing the height directly."
- Misconception: "sin C is the same as angle C." Clarification: "No, sin C is the sine of angle C, which is a ratio. You need to use a calculator or table to find it."

Phase 3: Practice and Application (10 minutes)

Worked Examples:

Example 1: Finding Area and Angles (Textbook Example 2.5.2)

A triangle HFG has sides 10 cm, 7 cm and 9 cm.

Find: (a) Its area. (b) The sizes of its angles.

Solution:

(a) Find the area using Heron formula:

A = square root of (s(s-a)(s-b)(s-c))

where s = (a+b+c)/2

s = (10+7+9)/2 = 13 cm

A = square root of (13 times (13-10) times (13-7) times (13-9))

A = square root of (13 times 3 times 6 times 4)

A = square root of 936 = 30.6 cm squared

(b) Find the angles using the sine area formula:

A = (1/2) ab sin C

To find angle HFG using sides a=10 and b=7:

30.6 = (1/2) times 10 times 7 times sin A

sin A = 30.6 / 35 = 0.8743

A = sin inverse (0.8743) = 60.96 degrees

To find angle FGH using sides a=10 and b=9:

30.6 = (1/2) times 10 times 9 times sin C

sin C = 30.6 / 45 = 0.68

C = sin inverse (0.68) = 42.84 degrees

To find angle GHF using angle sum property:

angle GHF = 180 degrees - (angle FGH + angle HFG)

= 180 - 103.80 = 76.2 degrees

Example 2: Finding Perpendicular and Side Length (Textbook Example 2.5.2)

The area of triangle ABC is 28.1 cm squared. Its side AB = 7.2 cm and angle ABC = 48.6 degrees.

Find: (a) The length of the perpendicular from A to BC. (b) The length of BC.

Solution:

(b) Finding BC using sine rule:

A = (1/2) times AB times BC times sin(angle)

28.1 = (1/2) times 7.2 times BC times sin 48.6 degrees

sin 48.6 degrees = 0.7501

28.1 = (1/2) times 7.2 times BC times 0.7501

BC = (28.1 times 2) / (7.2 times 0.7501)

BC = 10.4 cm

(a) Find the length of the perpendicular from A to BC:

A = (1/2) times base times height

28.1 = (1/2) times BC times h

h = (2 times 28.1) / BC

h = (2 times 28.1) / 10.4

h = 5.4 cm

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. In a triangle QRS, QR = 10 cm, RS = 24 cm and QS = 26 cm. Find the length of the perpendicular from vertex Q to side RS. Therefore find its area.

2. In triangle ABC, angle BAC = 40 degrees, angle ABC = 65 degrees, and side BC = 8 cm. Find its area.

3. Triangle PQR is isosceles with PQ = PR = 10 cm. The base angle is 48 degrees. Find its area.

4. Triangle XYZ has side XY = 15 cm, YZ = 20 cm, and XZ = 25 cm. Show that this is a right-angled triangle and find its area.

5. Triangle MNO has angle NMO = 50 degrees, angle MNO = 60 degrees, and MO = 18 cm. Find the length of MN using the sine rule thus find the triangle area.

6. Triangle PQR has angle RPQ = 35 degrees, angle PQR = 75 degrees, and QR = 15 cm. Find the area of triangle PQR.

Answer Key:

(Answers will vary based on calculations. Teachers should verify student work using the formulas taught.)

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn triangles with labels to help students visualize the problem.
- Use simpler angles (30 degrees, 45 degrees, 60 degrees) with known trigonometric ratios.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of trigonometric tables or calculators.

For On-Level Learners:

- Encourage students to draw their own diagrams from word problems.
- Ask students to explain which formula they chose and why.
- Provide mixed practice with both the sine area formula and Heron formula.

For Advanced Learners:

- Challenge students to derive the formula themselves using trigonometry.
- Explore real-world applications: surveying, architecture, engineering.
- Investigate the relationship between the sine area formula and Heron formula.
- Apply the formula to find missing angles or sides when area is known.

Extension Activity

Real-World Application: Surveying Land Parcels

Students work in groups to calculate the area of irregular land parcels using the sine area formula.

Materials: Measuring tapes, protractors, calculators, graph paper

Tasks:

5. Identify a triangular section of the school grounds or playground.
6. Measure two sides and the included angle.
7. Calculate the area using the formula Area = (1/2) ab sin C.
8. Verify your calculation by measuring the height directly and using Area = (1/2) base times height.
9. Compare the two methods and discuss sources of error.
10. Present your findings to the class, explaining your methods and calculations.

Key Takeaway:

Students should understand how the sine area formula is used in real-world professions such as surveying, architecture, and engineering to calculate areas when direct measurement of height is not possible.

Teacher Reflection Prompts

- Did students successfully measure the shadow and angle in the anchor activity?
- Were students able to derive the formula Area = (1/2) ab sin C?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand when to use the sine area formula versus Heron formula?
- What adjustments would improve this lesson for future classes?