CBC Grade 10 Mathematics Lesson Plan

Area of Octagons

Strand	Measurement and Geometry
Sub-Strand	Area of Polygons
Specific Learning Outcome	Explore the area of polygons as used in real-life situations, specifically calculating the area of regular octagons
Key Inquiry Questions	How do we calculate the area of regular octagons in real-life situations?
Learning Resources	CBC Grade 10 textbooks, compass, ruler, protractor, calculator, graph paper, string
Lesson Duration	40 minutes

Lesson Structure Overview

Phase	Activity	Duration
Phase 1	Problem-Solving and Discovery (Anchor Activity)	15 minutes
Phase 2	Structured Instruction (Key Takeaways)	10 minutes
Phase 3	Practice and Application (Worked Examples)	15 minutes
Phase 4	Assessment (Exit Ticket)	5 minutes

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Constructing and Analyzing a Regular Octagon

Work in Groups (2-3 students)

Materials:

- • Compass, ruler, protractor
- • Graph paper
- • String (optional for measuring circumradius)
- • Calculator

Tasks:

1. (a) Draw a circle with radius 6 cm on graph paper. This radius is called the circumradius (R), the distance from the center to any vertex of the octagon.

2. (b) Mark 8 equally spaced points around the circle. Since a full circle is 360°, each central angle between consecutive points should be 360° ÷ 8 = 45°. Use your protractor to mark points at 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°.
3. (c) Connect the 8 points to form a regular octagon. Verify that all sides appear equal in length.
4. (d) Draw lines from the center to each of the 8 vertices. This divides the octagon into 8 congruent (identical) triangles.
5. (e) Focus on one triangle. What is the vertex angle at the center? What is the length of the two equal sides (the radii)?
6. (f) Calculate the area of one triangle using the formula: A_triangle = (1/2) × R^2 × sin(45°). Remember that sin(45°) = √2/2.
7. (g) Multiply the area of one triangle by 8 to find the total area of the octagon.
8. (h) Share your results with classmates. Discuss: Why does dividing into triangles make the calculation easier?

Teacher Guidance for Anchor Activity

This hands-on activity helps students discover that regular polygons can be divided into congruent triangles radiating from the center, making area calculation systematic and manageable.

Facilitation Strategy:

- • Organize students into groups of 2-3
- • Demonstrate how to use a protractor to mark 45° intervals around the circle
- • Guide students to recognize that all 8 triangles are identical (congruent)
- • Emphasize that each triangle has two sides of length R (the circumradius) and a vertex angle of 45°
- • Ask: "What is the vertex angle for each triangle?"
- • Probe: "Why are all 8 triangles identical?"
- • Students should discover: Area of octagon = 8 × Area of one triangle

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

After students have explored through the anchor activity, formalize their discoveries with these key concepts:

1. Regular Octagons in Real-Life

Regular octagons (8-sided polygons with equal sides and angles) appear in many real-life contexts:

- • Architecture: Octagonal pavilions, gazebos, towers
- • Landscaping: Flower beds, garden features, paving patterns
- • Traffic: Stop signs (regular octagon shape)
- • Sports: Octagonal boxing rings, martial arts mats

- • Design: Tiles, decorative patterns, building layouts

2. Strategy: Divide into Triangles

The key strategy for finding the area of any regular polygon is to divide it into congruent triangles:

- • Draw lines from the center to each vertex
- • For an octagon, this creates 8 identical triangles
- • Each triangle has two sides equal to the circumradius R
- • The vertex angle at the center is 360° ÷ 8 = 45°

3. Formula for Area of One Triangle

Each triangle in the octagon has:

- • Two equal sides of length R (the circumradius)
- • Vertex angle θ = 45°

Using the formula for the area of a triangle with two sides and an included angle:

$A\_triangle = (1/2) \times R^2 \times \sin(45°)$

Since $\sin(45°) = \sqrt{2}/2$, we can simplify:

$A\_triangle = (1/2) \times R^2 \times (\sqrt{2}/2) = (R^2 \times \sqrt{2})/4$

4. Formula for Area of Regular Octagon

Since the octagon contains 8 identical triangles:

$A\_octagon = 8 \times A\_triangle$

$A\_octagon = 8 \times [(1/2) \times R^2 \times \sin(45°)]$

$A\_octagon = 4 \times R^2 \times \sin(45°)$

$A\_octagon = 4 \times R^2 \times (\sqrt{2}/2)$

$A\_octagon = 2R^2 \times \sqrt{2}$

Final Formula: $A = 2R^2\sqrt{2}$ (where R is the circumradius)

5. Understanding Circumradius

The circumradius R is the distance from the center of the octagon to any vertex. It is the radius of the circle that passes through all vertices of the octagon (the circumscribed circle).

6. Why This Formula Works

- • The octagon is divided into 8 congruent triangles
- • Each triangle has area $(1/2)R^2\sin(45°)$
- • Total area = $8 \times (1/2)R^2\sin(45°) = 4R^2\sin(45°) = 2R^2\sqrt{2}$
- • This method works for any regular polygon: divide by n, use angle 360°/n

Scaffolding Strategies

Address common challenges revealed during the anchor activity:

- • Emphasize that circumradius R is from center to vertex, not center to side
- • Use visual diagrams to show the 8 triangles clearly
- • Review $\sin(45°) = \sqrt{2}/2$ before calculations
- • Connect to special angles lesson (45° is a special angle)
- • Provide formula reference card: $A = 2R^2\sqrt{2}$

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Octagonal Flower Bed

A Mombasa school wants a regular octagonal flower bed, where the distance from the center to a vertex is 4 m. Calculate the area of the bed, giving your answer exactly in simplest surd form.

Solution:

Step 1: Identify the given information. The distance from center to vertex is the circumradius R = 4 m.

Step 2: Apply the octagon area formula. $A = 2R^2\sqrt{2}$

Step 3: Substitute R = 4 m.

$A = 2 \times (4)^2 \times \sqrt{2}$

$A = 2 \times 16 \times \sqrt{2}$

$A = 32\sqrt{2}\ m^2$

Step 4: Calculate approximate decimal value (optional).

$A \approx 32 \times 1.414 \approx 45.25\ m^2$

Answer: The area of the flower bed is $32\sqrt{2}$ m² (or approximately 45.25 m²).

Example 2: Octagonal Pavilion

A village wants to build an octagonal pavilion. The architect specifies that the circumradius should be 5 m. Calculate the area of the pavilion floor in simplest surd form.

Solution:

Step 1: Identify $R = 5$ m.

Step 2: Apply formula $A = 2R^2\sqrt{2}$.

Step 3: Substitute.

$A = 2 \times (5)^2 \times \sqrt{2}$

$A = 2 \times 25 \times \sqrt{2}$

$A = 50\sqrt{2}$ m²

Step 4: Approximate.

$A \approx 50 \times 1.414 \approx 70.7$ m²

Answer: The pavilion floor area is $50\sqrt{2}$ m² (or approximately 70.7 m²).

Example 3: Octagonal Stop Sign

A stop sign is a regular octagon with circumradius 30 cm. Calculate the area of the stop sign in square centimeters, giving your answer in simplest surd form.

Solution:

Step 1: $R = 30$ cm.

Step 2: $A = 2R^2\sqrt{2}$.

Step 3: Substitute.

$A = 2 \times (30)^2 \times \sqrt{2}$

$A = 2 \times 900 \times \sqrt{2}$

$A = 1800\sqrt{2}$ cm²

Step 4: Approximate.

$A \approx 1800 \times 1.414 \approx 2545.2$ cm²

Answer: The stop sign area is $1800\sqrt{2}$ cm² (or approximately 2545.2 cm²).

Example 4: Comparing Octagon and Circle

An octagonal garden bed and a circular garden bed both have the same circumradius of 3 m. Which has a larger area, and by how much?

Solution:

Step 1: Calculate octagon area. $A_{octagon} = 2R^2\sqrt{2} = 2 \times (3)^2 \times \sqrt{2} = 18\sqrt{2} \approx 25.46$ m²

Step 2: Calculate circle area. $A_{circle} = \pi R^2 = \pi \times (3)^2 = 9\pi \approx 28.27$ m²

Step 3: Compare. The circle has larger area.

Step 4: Find difference. Difference $= 9\pi - 18\sqrt{2} \approx 28.27 - 25.46 \approx 2.81$ m²

Answer: The circular bed has a larger area by approximately 2.81 m².

Individual Practice (Students work independently)

Provide students with similar problems to solve:

9. 1. A regular octagonal patio has a circumradius of 6 m. Calculate the area in simplest surd form.
10. 2. An octagonal boxing ring has a circumradius of 4.5 m. Find the area of the ring floor.
11. 3. A decorative octagonal tile has a circumradius of 8 cm. Calculate the area in simplest surd form.

Phase 4: Assessment - Exit Ticket (5 minutes)

Students complete individually to demonstrate understanding:

Question 1: A regular octagon has a circumradius of 7 m. Calculate the area in simplest surd form.

Question 2: An octagonal gazebo has a circumradius of 3.5 m. Find the approximate area to one decimal place.

Question 3: Explain why we divide a regular octagon into 8 triangles when calculating its area. What is the vertex angle of each triangle?

Exit Ticket Answer Key

Question 1:

$A = 2R^2\sqrt{2} = 2 \times (7)^2 \times \sqrt{2} = 2 \times 49 \times \sqrt{2} = 98\sqrt{2}\ m^2$

Question 2:

$A = 2 \times (3.5)^2 \times \sqrt{2} = 2 \times 12.25 \times \sqrt{2} = 24.5\sqrt{2} \approx 34.6\ m^2$

Question 3:

We divide the octagon into 8 triangles because each triangle is congruent (identical) and has two sides equal to the circumradius with a known vertex angle. This makes calculation systematic. The vertex angle is 360° ÷ 8 = 45°.

Differentiation Strategies

For Struggling Learners:

- • Provide pre-drawn octagons with triangles marked
- • Use formula reference cards: $A = 2R^2\sqrt{2}$
- • Review $\sin(45°) = \sqrt{2}/2$ before calculations
- • Allow calculators for $\sqrt{2}$ approximation
- • Break into explicit steps: identify R, apply formula, simplify

For Advanced Learners:

- • Derive the formula from first principles
- • Extend to other regular polygons (pentagon, hexagon, decagon)
- • Compare octagon area to inscribed/circumscribed circles
- • Explore relationship between side length and circumradius
- • Calculate apothem (perpendicular distance from center to side)

Real-World Connections

Area of octagons is essential in:

- • Architecture: Pavilions, gazebos, towers, building layouts
- • Landscaping: Flower beds, garden features, paving patterns
- • Traffic engineering: Stop sign design and manufacturing
- • Sports: Boxing rings, martial arts mats, sports facilities
- • Design: Tiles, decorative patterns, artistic layouts
- • Construction: Material estimation for octagonal structures

Post-Lesson Reflection for Teachers

- • Did students successfully divide the octagon into 8 congruent triangles?
- • Were students able to identify the circumradius correctly?
- • Did students apply the formula $A = 2R^2\sqrt{2}$ accurately?
- • What misconceptions emerged about circumradius vs. side length?
- • How engaged were students with the construction activity?
- • Did students express answers in simplest surd form?
- • What adjustments are needed for future lessons on regular polygons?