Grade 10 Mathematics Lesson Plan

Area of Sectors

Strand:	Measurement and Geometry
Sub-Strand:	Area of a Part of a Circle: Area of a Sector of a Circle
Specific Learning Outcome:	Work out the area of a sector of a circle in real-life situations. Apply the area of a part of a circle in real-life situations. Explore the use of the area of a part of a circle in real-life situations.
Duration:	40 minutes
Key Inquiry Question:	How do we use the concept of the area of a part of a circle in real life?
Learning Resources:	CBC Grade 10 textbooks, graph paper, razorblade or scissors, ruler, protractor, calculators

Lesson Structure Overview

Phase	Duration	Focus
Problem-Solving and Discovery	15 minutes	Anchor activity: Discovering the sector formula by cutting and measuring
Structured Instruction	10 minutes	Formalizing the sector formula and addressing misconceptions
Practice and Application	10 minutes	Worked examples and varied problems
Assessment	5 minutes	Exit ticket to check understanding

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Discovering the Sector Formula

Objective: Students will discover the formula for finding the area of a sector by cutting out a sector from a circle and exploring the relationship between the angle and the area.

Materials Needed:

- Graph paper
- Razorblade or a pair of scissors
- Ruler
- Protractor
- Calculator

Steps for the Activity:

1. Step 1: Draw a Circle. Draw a circle of radius 7 cm on a graph paper.
2. Step 2: Cut Out the Circle. Cut out the circle along its boundary.
3. Step 3: Mark the Centre. Mark the centre of the circle.
4. Step 4: Measure and Cut the Sector. Measure an angle of 30 degrees at the centre and cut out as shown.
5. Step 5: Estimate the Area. Estimate the area by counting the number of squares enclosed by the arc and the two radii of the circle.
6. Step 6: Express as a Fraction. Express the angle of the sector (30 degrees) as a fraction of the angle at the centre of the circle (360 degrees).
7. Step 7: Calculate. Multiply the fraction obtained in Step 6 by the area of the circle.
8. Step 8: Discuss and Share. Discuss and share the result with other groups.

Discussion Questions:

9. What is a sector?
10. How did you calculate the area of the sector?
11. What is the relationship between the angle and the area?
12. Did different groups get the same formula?

Teacher Role During Discovery:

- Circulate among groups, ensuring students understand how to measure the angle and cut the sector accurately.
- Ask probing questions: What fraction of the circle is this sector? How do we find the area?
- For struggling groups: Let us start by finding the area of the whole circle. What fraction of 360 degrees is 30 degrees? Now multiply the fraction by the area of the circle.
- For early finishers: Can you write a general formula for finding the area of any sector?
- Guide students to articulate: The area of a sector equals the fraction (angle/360) times the area of the circle.
- Identify 2-3 groups with clear findings to share with the class.

Phase 2: Structured Instruction (10 minutes)

Formalizing the Sector Formula and Addressing Misconceptions

After students have completed the anchor activity and shared their findings, the teacher formalizes the formula for finding the area of a sector.

Key Takeaway: What is a Sector?

A sector is a region bounded by two radii and an arc.

Minor sector is one whose area is less than a half of the area of the circle.

Major sector is one whose area is greater than a half of the area of the circle.

Formula:

Area of a Sector = (θ / 360) times π r squared

Where:

- θ is in degrees
- r is the radius of the circle
- π approximately equals 3.142 or 22/7

Scaffolding Strategies to Address Misconceptions:

- Misconception: I can use the angle directly without dividing by 360. Clarification: No, you must express the angle as a fraction of 360 degrees first.
- Misconception: The formula works for any angle unit. Clarification: No, the angle must be in degrees. If given in radians, convert first or use a different formula.
- Misconception: A sector is the same as a segment. Clarification: No, a sector includes the two radii, while a segment is the region between a chord and an arc.
- Misconception: I can use diameter instead of radius. Clarification: No, the formula uses radius. If given diameter, divide by 2 first.

Phase 3: Practice and Application (10 minutes)

Worked Example 1:

Find the area of a sector of a circle of radius 7 cm if the angle subtended at the centre is 90 degrees.

Solution:

The values given are, θ = 90 degrees, r = 7 cm

Area = (θ / 360) times π r squared

Area = (90 / 360) times (22 / 7) times (7 squared)

= (1 / 4) times (22 / 7) times 49

= (1 / 4) times 22 times 7

= 38.5 cm squared

Worked Example 2:

Find the area of a sector of a circle where θ = 45 degrees, r = 10 cm (use π = 3.142).

Solution:

Area = (θ / 360) times π r squared

Area = (45 / 360) times 3.142 times (10 squared)

= (1 / 8) times 3.142 times 100

= 39.275 cm squared

Worked Example 3: Windscreen Wiper Problem

The shaded region shows the area swept out on a flat windscreen by a wiper. The larger sector has radius 20 cm and angle 120 degrees. The smaller sector has radius 16 cm and angle 120 degrees. Calculate the area of this region.

Solution:

The area of the region is gotten by subtracting the Area of the smaller sector from Area of the larger sector.

Area of the larger sector:

R = 20 cm, θ = 120 degrees

A = (120 / 360) times (22 / 7) times 20 squared

= (1 / 3) times (22 / 7) times 400

= 419.05 cm squared

Area of the smaller sector:

r = 16 cm, θ = 120 degrees

A = (120 / 360) times (22 / 7) times 16 squared

= (1 / 3) times (22 / 7) times 256

= 268.19 cm squared

Therefore:

Area of the region = 419.05 minus 268.19 = 150.86 cm squared

Phase 4: Assessment (5 minutes)

Exit Ticket:

Students complete the following questions individually.

1. A sector of a circle of radius r is subtended at the centre by an angle of θ. Calculate the area of the sector if: (a) r = 10 m, θ = 264 degrees (b) r = 8.4 cm, θ = 40 degrees (c) r = 1.4 cm, θ = 80 degrees

2. A sector has an angle of π/3 radians and a radius of 8 cm. Find its area.

3. A goat is tethered at the corner of a fenced rectangular grazing field. If the length of the rope is 21 m, what is its grazing area?

Answer Key:

1. (a) A = (264 / 360) times π times 10 squared = (264 / 360) times (22 / 7) times 100 = 733.33 m squared. (b) A = (40 / 360) times π times 8.4 squared = (40 / 360) times (22 / 7) times 70.56 = 24.64 cm squared. (c) A = (80 / 360) times π times 1.4 squared = (80 / 360) times (22 / 7) times 1.96 = 1.37 cm squared.

2. For radians, use A = (1 / 2) times r squared times θ. A = (1 / 2) times 8 squared times (π / 3) = (1 / 2) times 64 times (π / 3) = 33.51 cm squared.

3. The goat can graze in a quarter circle (90 degrees) with radius 21 m. A = (90 / 360) times π times 21 squared = (1 / 4) times (22 / 7) times 441 = 346.5 m squared.

Differentiation Strategies

For Struggling Learners:

- Provide pre-drawn circles with sectors already outlined.
- Use simpler angles (e.g., 90 degrees, 180 degrees) for initial practice.
- Pair struggling students with confident problem solvers.
- Provide step-by-step calculation templates.
- Allow use of calculators.
- Break down the formula into steps: Find the fraction, find the area of the circle, multiply.

For On-Level Learners:

- Encourage students to verify their formula with different angles.
- Ask students to explain the relationship between the angle and the area.
- Provide mixed practice with different types of sector problems.
- Challenge students to solve problems involving major sectors.

For Advanced Learners:

- Challenge students to derive the formula for sectors in radians.
- Explore real-world applications: pizza slices, pie charts, windscreen wipers, grazing areas.
- Investigate the relationship between sector area and arc length.
- Solve optimization problems: Given a fixed radius, what angle maximizes or minimizes the sector area?
- Apply the concept to composite shapes involving multiple sectors.

Extension Activity

Real-World Application: Pizza Slice Design Project

Individual or Group Work

Situation: A pizza restaurant wants to design different pizza slice sizes for their customers.

Task:

13. Design a circular pizza with radius 30 cm.
14. Calculate the area of one slice if the pizza is cut into 8 equal slices.
15. Calculate the area of one slice if the pizza is cut into 6 equal slices.
16. If a customer wants a slice that is 50 cm squared, what angle should the slice have?
17. Present your findings with diagrams and calculations.

Key Takeaway:

Students should understand how the area of a sector is used in real-world contexts such as food service, agriculture (grazing areas), engineering (windscreen wipers), and data visualization (pie charts).

Teacher Reflection Prompts

- Did students successfully discover the sector formula through the anchor activity?
- Were students able to cut and measure the sector accurately?
- What misconceptions emerged during the lesson, and how were they addressed?
- Did students understand the difference between a sector and a segment?
- What adjustments would improve this lesson for future classes?