CBC Grade 10 Mathematics Lesson Plan

Area of a Part of a Circle in Real-Life

Strand	**Measurement and Geometry**
Sub-Strand	Area of a Part of a Circle
Specific Learning Outcome	Apply formulas for sectors, segments, and annuli to solve real-life problems involving pizza slicing, washers, and signal coverage
Key Inquiry Questions	How do we calculate the area of parts of circles in everyday situations?
Learning Resources	CBC Grade 10 textbooks, orange, knife, ruler, plate, calculator
Lesson Duration	40 minutes

Lesson Structure Overview

Phase	**Activity**	**Duration**
Phase 1	Problem-Solving and Discovery (Anchor Activity)	15 minutes
Phase 2	Structured Instruction (Key Takeaways)	10 minutes
Phase 3	Practice and Application (Worked Examples)	15 minutes
Phase 4	Assessment (Exit Ticket)	5 minutes

Phase 1: Problem-Solving and Discovery (15 minutes)

Anchor Activity: Investigating Orange Slices

Work in Groups (2-3 students)

Materials:

- • An orange and a knife
- • A ruler, pen, and book to record measurements
- • A plate
- • Calculator

Tasks:

1. (a) Cut the orange into two equal halves crossectionally to obtain a circular cross-section.

2. (b) From the crossectional cuts, slice them to obtain 5 slices of the orange, each slice being a part of a circle (annulus shape with juice sacs in the middle and peel on the outside).
3. (c) Take one slice and measure: (i) Internal radius (from center to where juice sac ends), (ii) External radius (from center to the exocarp/peel).
4. (d) Repeat measurements for the other slices and record all data.
5. (e) Calculate the area of each annulus (the edible part between inner and outer circles) using the formula: $A_{annulus} = \pi(R^2 - r^2)$, where R is the outer radius and r is the inner radius.
6. (f) Share your results with classmates and compare answers. Discuss: Why do we subtract the inner circle area from the outer circle area?

Teacher Guidance for Anchor Activity

This hands-on activity helps students discover the concept of annulus (ring-shaped region) through a familiar object—an orange slice. By measuring and calculating the edible area, students connect abstract formulas to tangible real-world objects.

Facilitation Strategy:

- • Organize students into groups of 2-3
- • Demonstrate safe cutting technique or pre-cut oranges for safety
- • Guide students to identify the two radii: inner (juice sac boundary) and outer (peel edge)
- • Encourage accurate measurement using rulers
- • Ask: "What shape is the edible part of the orange slice?"
- • Probe: "Why do we subtract the inner area from the outer area?"
- • Students should discover: Annulus area = Outer circle - Inner circle

Phase 2: Structured Instruction (10 minutes)

Key Takeaways

After students have explored through the anchor activity, formalize their discoveries with these key concepts:

1. Parts of a Circle in Real-Life

Area of part of a circle refers to calculating the area of sectors, segments, or annuli. Real-life applications include:

- • Bakers: Calculate pizza slice area to determine portion sizes
- • Pastry chefs: Calculate cake slice area for serving
- • Fruit sellers: Calculate orange/watermelon slice area for pricing
- • Engineers: Design washers, gears, and mechanical parts
- • Telecommunications: Model overlapping signal coverage areas

2. Three Key Formulas

Shape	Formula	Real-Life Example
Sector	$A = (\theta/360) \times \pi r^2$	Pizza slice, cake slice
Annulus	$A = \pi(R^2 - r^2)$	Washer, orange slice, ring
Segment	A = A_sector - A_triangle	Signal overlap, window design

3. Sector: Pizza Slicing

A sector is a "pie-shaped" region bounded by two radii and an arc. When a pizza is sliced into equal pieces, each slice is a sector.

Formula: $A_{sector} = (\theta/360) \times \pi r^2$

Where θ is the central angle in degrees and r is the radius.

Key insight: The fraction $\theta/360$ represents what portion of the full circle the sector occupies.

4. Annulus: Metal Washers and Orange Slices

An annulus is a ring-shaped region between two concentric circles (circles with the same center).

Formula: $A_{annulus} = \pi(R^2 - r^2) = \pi R^2 - \pi r^2$

Where R is the outer radius and r is the inner radius.

Key insight: Subtract the inner "hole" area from the outer circle area to get the ring area.

Applications: Metal washers distribute load in bolts/nuts; orange slices have edible ring between juice sac and peel.

5. Segment: Signal Coverage Overlap

A segment is a region bounded by a chord and an arc. It is calculated by subtracting the triangle area from the sector area.

Formula: $A_{segment} = A_{sector} - A_{triangle}$

Where $A_{sector} = (\theta/360) \times \pi r^2$ and $A_{triangle} = (1/2)r^2\sin(\theta)$

Key insight: The segment is the "leftover" area after removing the triangle from the sector.

Applications: Overlapping signal ranges from towers, architectural window designs.

6. Choosing the Right Formula

- • Sector: When the region is bounded by two radii and an arc (pizza slice, cake slice)
- • Annulus: When the region is a ring between two circles (washer, orange slice)
- • Segment: When the region is bounded by a chord and an arc (signal overlap, window)

Scaffolding Strategies

Address common challenges revealed during the anchor activity:

- • Emphasize visual identification: sector = pie slice, annulus = ring, segment = chord region
- • Use diagrams to show which formula applies to which shape
- • Connect formulas to subtraction logic (annulus and segment both involve subtraction)
- • Provide formula reference chart for quick lookup
- • Use real objects (pizza, washer, orange) to reinforce concepts

Phase 3: Practice and Application (15 minutes)

Worked Examples

Example 1: Slicing a Pizza (Sector)

A pizza has a radius of 12 inches and is sliced into 8 equal pieces. Calculate the area of each slice of pizza.

Solution:

Step 1: Identify the shape. Each pizza slice is a sector.

Step 2: Determine the central angle. Since the pizza is sliced into 8 equal pieces, each slice corresponds to $\theta = 360°/8 = 45°$.

Step 3: Apply the sector formula. $A\_sector = (\theta/360) \times \pi r^2$

$A\_sector = (45/360) \times \pi(12)^2 = (1/8) \times \pi(144) = (1/8) \times (22/7) \times 144$

$A\_sector = (1/8) \times 452.57 = 56.57\ in^2$

Answer: Each pizza slice has an area of approximately 56.57 square inches.

Example 2: Designing a Metal Washer (Annulus)

A metal washer has an outer radius of 5 cm and an inner radius of 2 cm. Calculate the area of the washer.

Solution:

Step 1: Identify the shape. A washer is an annulus (ring).

Step 2: Identify the radii. Outer radius $R = 5$ cm, Inner radius $r = 2$ cm.

Step 3: Apply the annulus formula. $A\_annulus = \pi(R^2 - r^2)$

$A\_annulus = \pi(5^2 - 2^2) = \pi(25 - 4) = \pi(21)$

$A\_annulus = (22/7) \times 21 = 66 \text{ cm}^2$

Answer: The area of the washer is 66 square centimeters.

Example 3: Modelling Overlapping Signal Ranges (Segment)

Two towers have signal ranges that overlap. The radius of the signal range of the first tower is 10 km and the central angle of the overlapping area is 60°. Calculate the area of the overlapping signal range (segment).

Solution:

Step 1: Identify the shape. The overlapping area is a segment.

Step 2: Calculate the sector area. $A\_sector = (\theta/360) \times \pi r^2$

$A\_sector = (60/360) \times \pi(10)^2 = (1/6) \times \pi(100) = (100\pi/6) \approx 52.36 \text{ km}^2$

Step 3: Calculate the triangle area. $A\_triangle = (1/2)r^2\sin(\theta)$

$A\_triangle = (1/2)(10)^2\sin(60°) = 50 \times (\sqrt{3}/2) = 25\sqrt{3} \approx 43.30 \text{ km}^2$

Step 4: Calculate the segment area. A_segment = A_sector - A_triangle

$A\_segment = 52.36 - 43.30 = 9.06 \text{ km}^2$

Answer: The area of the overlapping signal range is approximately 9.06 square kilometers.

Example 4: Circular Garden Path (Annulus)

A circular garden has a radius of 10 meters. A circular path with a width of 2 meters surrounds the garden. Calculate the area of the path.

Solution:

Step 1: Identify the shape. The path is an annulus.

Step 2: Determine the radii. Inner radius (garden) r = 10 m, Outer radius (garden + path) R = 10 + 2 = 12 m.

Step 3: Apply the annulus formula. $A\_annulus = \pi(R^2 - r^2)$

$A\_annulus = \pi(12^2 - 10^2) = \pi(144 - 100) = \pi(44)$

$A\_annulus = (22/7) \times 44 = 138.29\ m^2$

Answer: The area of the path is approximately 138.29 square meters.

Individual Practice (Students work independently)

Provide students with similar problems to solve:

7. 1. A pizza has a radius of 14 inches and is sliced into 10 equal pieces. Calculate the area of each slice.
8. 2. A washer has an outer radius of 6 cm and an inner radius of 3 cm. Calculate the area of the washer.
9. 3. A circular swimming pool has a radius of 8 meters. A circular deck with a width of 3 meters surrounds the pool. Calculate the area of the deck.

Phase 4: Assessment - Exit Ticket (5 minutes)

Students complete individually to demonstrate understanding:

Question 1: A circular cake has a radius of 15 cm and is sliced into 12 equal pieces. Calculate the area of each slice.

Question 2: A metal ring has an outer radius of 8 cm and an inner radius of 5 cm. Calculate the area of the ring.

Question 3: Explain the difference between a sector, an annulus, and a segment. Give one real-life example for each.

Exit Ticket Answer Key

Question 1:

$\theta = 360°/12 = 30°$

$A\_sector = (30/360) \times \pi(15)^2 = (1/12) \times \pi(225) = (1/12) \times 706.86 = 58.91\ cm^2$

Question 2:

$A\_annulus = \pi(8^2 - 5^2) = \pi(64 - 25) = \pi(39) = (22/7) \times 39 = 122.57\ cm^2$

Question 3:

• Sector: Pie-shaped region bounded by two radii and an arc. Example: Pizza slice, cake slice.

• Annulus: Ring-shaped region between two concentric circles. Example: Washer, orange slice.

• Segment: Region bounded by a chord and an arc. Example: Signal overlap, architectural window.

Differentiation Strategies

For Struggling Learners:

- • Provide formula reference cards with visual diagrams
- • Use physical models (pizza slices, washers, orange slices)
- • Break calculations into explicit steps
- • Focus on one shape type (sector) initially before introducing others
- • Provide worked example templates to follow

For Advanced Learners:

- • Challenge with annular sectors (combination of annulus and sector)
- • Explore optimization problems (maximize area with constraints)
- • Connect to careers (engineering, architecture, telecommunications)
- • Investigate common regions between overlapping circles
- • Derive formulas from first principles

Real-World Connections

Area of parts of circles is essential in:

- • Food industry: Pizza slicing, cake portioning, fruit pricing
- • Manufacturing: Washer design, gear design, mechanical parts
- • Telecommunications: Signal coverage modeling, tower placement
- • Architecture: Window design, decorative elements
- • Agriculture: Irrigation coverage, circular field sections
- • Engineering: Load distribution, structural design

Post-Lesson Reflection for Teachers

- • Did students successfully distinguish between sectors, annuli, and segments?
- • Were students able to apply the correct formula for each shape?
- • What misconceptions emerged about the formulas or shapes?
- • How engaged were students with the orange slicing activity?
- • Did students connect the concepts to real-world applications?
- • What adjustments are needed for future lessons on this topic?